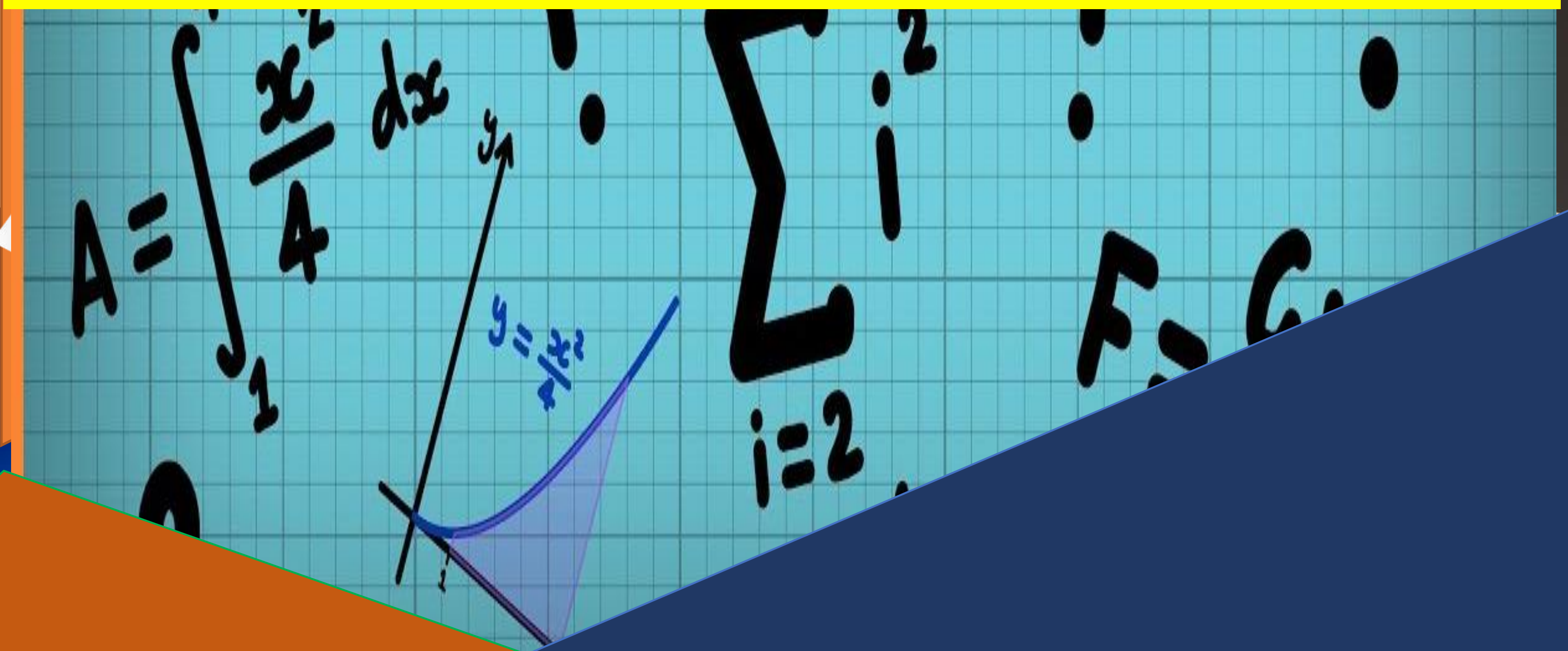
MATHEMATICS



CLASS-11

TRIGONOMETRY

e Shiksha Pie
Acharya

ASPIRING YOUNG MINDS

E SHIKSHA PIE PROGRAMS

CLASSES VI, VII, VIII, IX, X, XI AND XII

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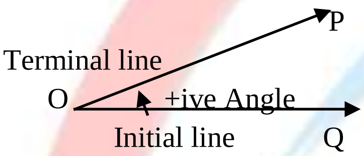
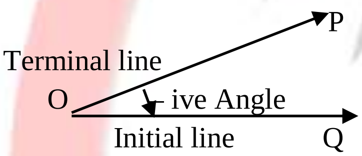
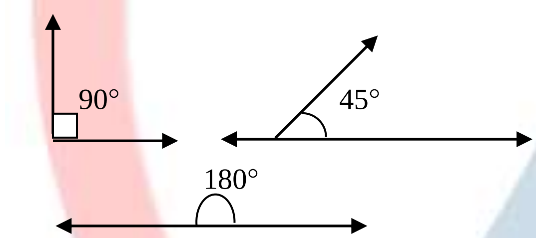
TRIGONOMETRIC IDENTITIES

ANGLES

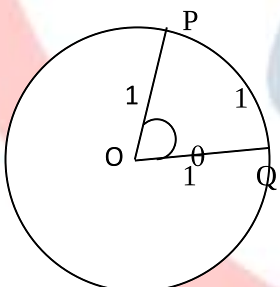




ANGLE MEASURE(In degrees)



RADIAN MEASURE



θ = Arc/Radius

= 1/1 radian = 1 radian = 1^c.

Area of sector = $\frac{1}{2}r^2\theta$.

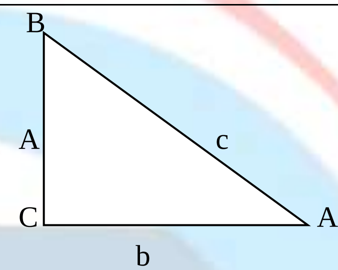
GENERAL DEFINITION OF TRIGONOMETRIC RATIOS

In a right ΔABC, $\angle$ B = 90°. AB = c, BC = a and CA = b. By the given Δ ABC if $\angle$A = θ, then: sinθ = per/hy = a/b,

cosθ = base/hy = c/b, tan θ = per/base = a/c,

cotθ = base/per = c/a,

secθ = hy/base = b/c, cosecθ = hy/per = b/a



RELATION BETWEEN DEGREE AND RADIAN

π = 180° $\Rightarrow$ π/2 = 90°,

Use π = 22/7, we get: 1° = π/180 rad

1° = 0.01746 radian,

So, 1 rad = 180/π = 180 × 7/22 = 57°16' (Approx)

RELATION BETWEEN TRIGONOMETRIC RATIOS

sinA = 1/cosecA, cosecA = 1/sinA,

cosA = 1/secA, secA = 1/cosA,

tanA = 1/cotA, cotA = 1/tanA.

SIGN OF TRIGONOMETRIC RATIOS

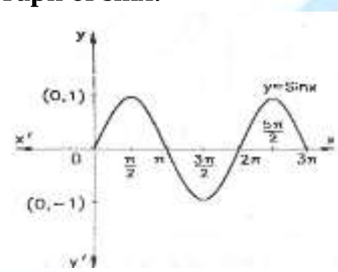
(a) In Q-1, x and y are + ive so, all T-Ratios are + ive.

(b) In Q-2, x is – ive and y is + ive so, only sin and cosec are + ive.

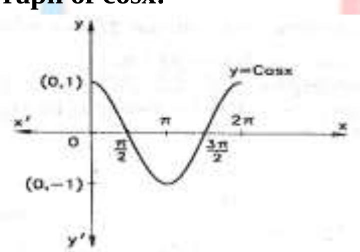
(c) In Q-3, x and y both are – ive so, only tan and cot are + ive.

(d) In Q-4, x is + ive and y is – ive so, only cos and sec is + ive.

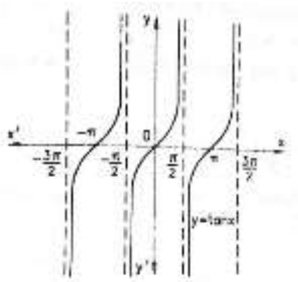
Graph of sinx:



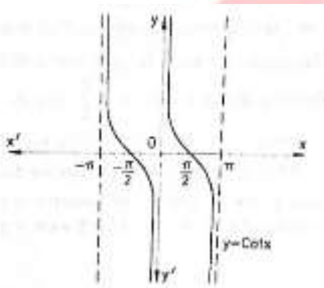
Graph of cosx:



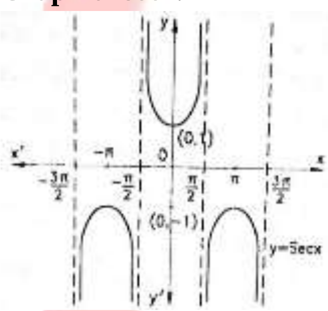
Graph of tanx:



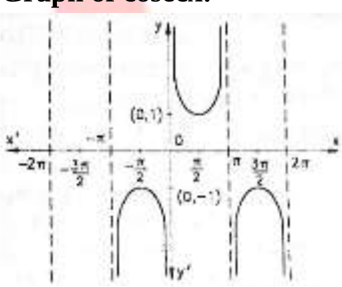
Graph of cotx:



Graph of secx:



Graph of cosecx:



BASIC FORMULAS

(a) $\sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta$

$\Rightarrow \cos^2\theta = 1 - \sin^2\theta$

(b) $1 + \tan^2\theta = \sec^2\theta \Rightarrow \sec^2\theta - 1 = \tan^2\theta$

$\Rightarrow \sec^2\theta - \tan^2\theta = 1$

(c) $1 + \cot^2\theta = \mathrm{cosec}^2\theta$

$\Rightarrow \mathrm{cosec}^2\theta - 1 = \cot^2\theta$

$\Rightarrow \mathrm{cosec}^2\theta - \cot^2\theta = 1$

(d) $\tan\theta = \sin\theta/\cos\theta$ and $\cot\theta = \cos\theta/\sin\theta$

(e) For $0 < A < \pi/2$, $0 < \cos A < \frac{\sin A}{A} < \frac{1}{\cos A}$

(f) (i) $\sin\theta = 1/\sqrt{(1 + \cot^2\theta)}$,

(ii) $\sec\theta = \sqrt{(1 + \tan^2\theta)}$,

(iii) $\cos\theta = \cot\theta/\sqrt{(1 + \cot^2\theta)}$,

(iv) $\tan\theta = 1/\cot\theta$.

deg	0^0	30°	45°	60°	90°
Rad	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sin	0	½	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	½	0
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$\propto$

RATIOS OF ALLIED ANGLES

{Change for $\pi/2$, $3\pi/2$ and no change for $-\theta$, π, 2π}

NOTE: Reciprocals of T-R will follow the same rule.

(a) T-R for ($2\pi n + \theta$)

(i) $\sin(2n\pi + \theta) = \sin\theta$

(ii) $\cos(2n\pi + \theta) = \cos\theta$

(iii) $\tan(2n\pi + \theta) = \tan\theta$

(b) T-R for ($\pi/2 \pm \theta$)

(i) $\sin(\pi/2 + \theta) = \cos\theta$, $\sin(\pi/2 - \theta) = \cos\theta$

(ii) $\cos(\pi/2 + \theta) = -\sin\theta$, $\cos(\pi/2 - \theta) = \sin\theta$

(iii) $\tan(\pi/2 + \theta) = -\cot\theta$, $\tan(\pi/2 - \theta) = \cot\theta$

(c) T-R for ($\pi \pm \theta$)

(i) $\sin(\pi + \theta) = -\sin\theta$, $\sin(\pi - \theta) = \sin\theta$

(ii) $\cos(\pi + \theta) = -\cos\theta$, $\cos(\pi - \theta) = -\cos\theta$

(iii) $\tan(\pi + \theta) = \tan\theta$, $\tan(\pi - \theta) = -\tan\theta$

(d) T-R for ($3\pi/2 \pm \theta$)

(i) $\sin(3\pi/2 + \theta) = -\cos\theta$, $\sin(3\pi/2 - \theta) = -\cos\theta$

(ii) $\cos(3\pi/2 + \theta) = \sin\theta$, $\cos(3\pi/2 - \theta) = -\sin\theta$

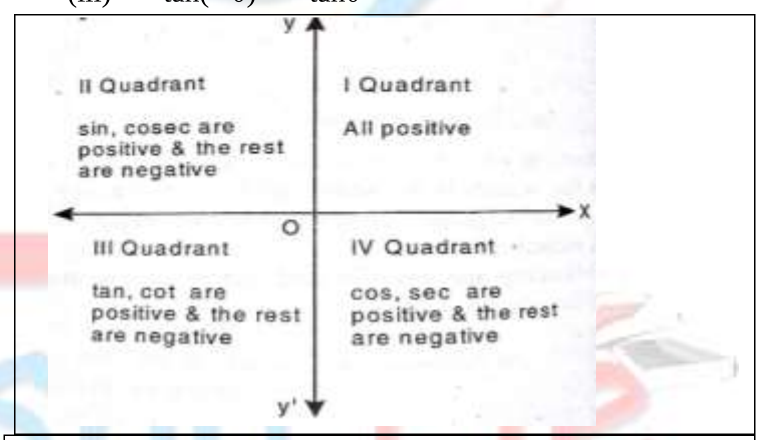
(iii) $\tan(3\pi/2 + \theta) = -\cot\theta$, $\tan(3\pi/2 - \theta) = \cot\theta$

(e) T-R for ($-\theta$)

(i) $\sin(-\theta) = -\sin\theta$

(ii) $\cos(-\theta) = \cos\theta$

(iii) $\tan(-\theta) = -\tan\theta$



VALUES OF TRIGONOMERTRIC RATIOS

(a) $\sin\theta$ and $\cos\theta$ never greater than 1 and never less than -1.

(b) $\sec\theta$ and $\mathrm{cosec}\,\theta$ never lie between -1 and $+1$.

(c) $\tan\theta$ and $\cot\theta$ both can take any real value between $-\infty$ and $+\infty$.

Some other values

(a) $\sin 15^\circ = (\sqrt{3}-1)/2\sqrt{2} = \cos 75^\circ$

(b) $\cos 15^\circ = (\sqrt{3}+1)/2\sqrt{2} = \sin 75^\circ$

(c) $\tan 15^\circ = 2-\sqrt{3} = \cot 75^\circ$

(d) $\cot 15^\circ = 2+\sqrt{3} = \tan 75^\circ$

(e) $\sin 22\frac{1}{2} = \frac{1}{2}\sqrt{(2-\sqrt{2})}$

(f) $\cos 22\frac{1}{2} = \frac{1}{2}\sqrt{(2+\sqrt{2})}$

(g) $\tan 22\frac{1}{2} = \sqrt{2}-1$

(h) $\cot 22\frac{1}{2} = \sqrt{2}+1$

(i) $\sin 18^\circ = (\sqrt{5}-1)/4 = \cos 72^\circ$

(j) $\cos 18^\circ = \sqrt{(10+2\sqrt{5})}/4 = \sin 72^\circ$

(k) $\sin 36^\circ = \sqrt{(10-2\sqrt{5})}/4 = \cos 54^\circ$

(l) $\cos 36^\circ = (\sqrt{5}+1)/4 = \sin 54^\circ$

T.R. for sum and difference of angles and other formulae

(a) $\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$

(b) $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

(c) $\tan(A\pm B) = \{\tan A\pm\tan B\}/\{1\mp\tan A\tan B\}$

(d) $\cot(A\pm B) = \{\cot A\cot B\mp 1\}/\{\cot B\pm\cot A\}$

(e) $\sin 2\theta = 2\sin\theta\cos\theta = 2\tan\theta/\{1+\tan^2\theta\}$

(f) $\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$= \cos^2\theta - \sin^2\theta = \{1-\tan^2\theta\}/\{1+\tan^2\theta\}$

(g) $\tan 2\theta = 2\tan\theta/(1-\tan^2\theta)$

(h) $\cot 2\theta = \{\cot^2\theta - 1\}/2\cot\theta$

(i) $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$

(j) $\cos 3\theta = 4\cos^3\theta - 3\cos\theta$

(k) $\tan 3\theta = \{3\tan\theta - \tan^3\theta\}/\{1-3\tan^3\theta\}$

(l) $\cot 3\theta = \{\cot^3\theta - 3\cot\theta\}/\{3\cot^2\theta - 1\}$

(m) $\sin(A+B+C) = \sin A\cos B\cos C + \cos A\sin B\cos C$

$+ \cos A\cos B\sin C - \sin A\sin B\sin C$

(n) $\cos(A+B+C) = \cos A\cos B\cos C - \cos A\sin B\sin C$

$- \sin A\cos B\sin C - \sin A\sin B\cos C$

(o) $\tan(A+B+C) = \frac{\tan A+\tan B+\tan C-\tan A.\tan B.\tan C}{1-\tan A.\tan B-\tan B.\tan C-\tan C.\tan A}$

Some other important formulas

(a) $\sin(A+B).\sin(A-B) = \sin^2 A - \sin^2 B = \cos^2 B - \cos^2 C$

(b) $\cos(A+B).\cos(A-B) = \cos^2 B - \sin^2 A = \cos^2 A - \sin^2 B$

(c) $(1-\cos\theta)/\sin\theta = \tan\theta/2 = \sin\theta/(1+\cos\theta)$

(d) $\tan(\pi/4\pm\theta) = (1\pm\tan\theta)/(1\mp\tan\theta)$

$= (\cos\theta\pm\sin\theta)/(\cos\theta+\sin\theta)$

$= (1\pm\sin\theta)/\cos 2\theta$

(e) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(f) $2\sin B\cos A = \sin(A+B) - \sin(A-B)$

(g) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(h) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(i) $\sin C + \sin D = 2\sin(C+D)/2.\cos(C-D)/2$

(j) $\sin C - \sin D = 2\cos(C+D)/2.\sin(C-D)/2$

(k) $\cos C + \cos D = 2\cos(C+D)/2.\cos(C-D)/2$

(l) $\cos C - \cos D = 2\sin(C+D)/2.\sin(D-C)/2$

(m) $\tan(A\pm B) = \sin(A\pm B)/\cos A.\cos B$

(n) $\cot A\pm\cot B = \sin(B\pm A)/\sin A.\sin B$

(o) $\sin A/2 = \pm\sqrt{(1-\cos A)/2}$

(p) $\cos A/2 = \pm\sqrt{(1+\cos A)/2}$

(q) $\tan A/2 = \pm\sqrt{(1-\cos A)(1+\cos A)}$

(r) $\cot A/2 = \pm\sqrt{(1+\cos A)(1-\cos A)}$

SOME USEFUL RESULTS

(a) If $A+B+C=\pi$, then

(i) $\sin 2A + \sin 2B + \sin 2C = 4\sin A.\sin B.\sin C$

(ii) $\cos 2A + \cos 2B + \cos 2C = -1 - 4\cos A.\cos B.\cos C$.

(iii) $\cos A + \cos B + \cos C = 1 + 4\sin A/2\sin B/2\sin C/2$

(iv) $\sin A + \sin B + \sin C = 4\cos A/2\cos B/2\cos C/2$

(v) $\tan A + \tan B + \tan C = \tan A\tan B\tan C$

(vi) $\cot A\cot B + \cot B\cot C + \cot C\cot A = 1$

(vii) $\tan A/2\tan B/2 + \tan B/2\tan C/2 + \tan C/2\tan A/2 = 1$

(viii) $\tan A\tan B + \tan B\tan C + \tan C\tan A = 1$,

(b) If $A+B+C=\pi/2$

(ix) $\cot A/2 + \cot B/2 + \cot C/2 = \cot A/2\cot B/2\cot C/2$

(x) $\sin^2 A + \sin^2 B + \sin^2 C = 2 + 2\cos A\cos B\cos C$

(xi) $\cos^2 A + \cos^2 B + \cos^2 C = 1 - 2\cos A\cos B\cos C$

(xii) $\sin^2 A/2 + \sin^2 B/2 + \sin^2 C/2$

$= 1 - 2\sin A/2\sin B/2\sin C/2$

(xiii) $\cos^2 A/2 + \cos^2 B/2 + \cos^2 C/2$

$= 2 + 2\sin A/2\sin B/2\sin C/2$

(b) Some useful results

(i) $\sin\theta.\sin(60^\circ-\theta).\sin(60^\circ+\theta) = \{\sin 3\theta\}/4$

(ii) $\cos\theta.\cos(60^\circ-\theta).\cos(60^\circ+\theta) = \{\cos 3\theta\}/4$

(iii) $\tan\theta\,.\,\tan(60^\circ-\theta)\,.\,\tan(60^\circ+\theta) = \tan 3\theta$

(iv) $\sin\alpha + \sin\beta + \sin\gamma - \sin(\alpha+\beta+\gamma)$

$= 4\sin(\alpha+\beta)/2.\sin(\beta+\gamma)/2.\sin(\gamma+\alpha)/2$

(v) $\cos\alpha + \cos\beta + \cos\gamma + \cos(\alpha+\beta+\gamma)$

$= 4\cos(\alpha+\beta)/2.\cos(\beta+\gamma)/2.\cos(\gamma+\alpha)/2$

(vi) $\sin a + \sin(a+d) + \sin(a+2d)$

$+ \ldots + \sin\{a+(n-1)d\}$

$= [\sin(nd/2)/(\sin d/2)] \times [\sin\{2a+(n-1)d\}/2]$

(vii) $\cos a + \cos(a+d) + \cos(a+2d)$

$+\ldots + \cos\{a+(n-1)d\}$

$= [\sin(nd/2)/(\sin d/2)] \times [\cos\{2a+(n-1)d\}/2]$

(viii) $\cos A.\cos 2A.\cos 2^2A\ldots\ldots\cos 2^{n-1}A = \sin 2^nA/2^n\sin A$.

(ix) $\sin A/2 \pm \cos A/2 = \sqrt{(1\pm\sin A)}$

(x) $\tan A/2 = \pm\{\sqrt{(1+\tan^2 A)} - 1\}/\tan A$

(xi) $|a\cos A + b\sin A| \le \sqrt{(a^2 + b^2)}$

(xii) $\cos A \pm \sin A = \sqrt{2}\sin(\pi/4 \pm A) = \sqrt{2}\cos(A \mp \pi/4)$

MAXIMUM AND MINIMUM VALUE OF a cosA + b sinA

(a) max. value of acosA + bsinA = $+\sqrt{(a^2 + b^2)}$

(b) min. value of acosA + bsinA = $-\sqrt{(a^2 + b^2)}$

MAXIMUM AND MINIMUM VALUE OF acosA + bsinA ± c

(a) max. value of acosA + bsinA ± c = $+\sqrt{(a^2 + b^2)} \pm c$.

(b) min. value of acosA + bsinA ± c = $-\sqrt{(a^2 + b^2)} \pm c$.

EXAMPLE –1

Find the number of minutes and seconds in 4.5 degree.

Solution:

(1) 4.5 degree = 4.5 × 60 = 270 min.

(2) 4.5degree = 270 × 60 = 16,200 sec.

EXAMPLE – 2

Express 60°, 75°, 115° into radian.

Solution:

$180° = \pi \Rightarrow 1° = \pi/180$ rad.

So, $60° = \pi/180 \times 60 = \pi/3$.

$75° = \pi/180 \times 75 = 5\pi/12$

$115° = \pi/180 \times 115 = 23\pi/36$. Ans.

EXAMPLE – 3

Express $4\pi/9$, $7\pi/12$, 6 radians in degrees.

Solution:

$\pi = 180° \Rightarrow 1$ rad $= 180°/\pi$

$4\pi/9 = 4\pi/9 \times 180°/\pi = 80°$

$7\pi/12 = 7\pi/12 \times 180°/\pi = 105°$

6 rad $= 180/\pi \times 6 = 180/22 \times 7 \times 6 = 343°\ 38'(10\frac{10}{11})''$ Ans.

EXAMPLE – 4

The angles of a quadrilateral are in AP and the greatest angle is 120°. Find the angles in radian.

Solution:

Let the angles are (a – 3d) , (a – d), (a + d) and (a + 3d).

Now, a + 3d = 120°………(1) And

(a – 3d) + (a – d) + (a + d) + (a + 3d) = 360°

$\Rightarrow 4a = 360° \Rightarrow a = 90°$ and d = 10°

Hence, Angle A = 60° = $\pi/3$,

$\angle B = 80° = 4\pi/9$, $\angle C = 100° = 5\pi/9$, $\angle D = 120° = 2\pi/3$.

EXAMPLE – 5

Find the length of an arc of circle of radius 10 cm subtends an angle of 45°at the center.

Solution:

Let $\angle A = 45°$, and radius = 10 cm.

Angle A = arc/radius $\Rightarrow 45° = \pi/4$ = arc/radius

$\Rightarrow \pi/4$ = arc/10 $\Rightarrow$ arc = $5\pi/2$ cm.

EXAMPLE – 6

If the arcs of the same length of two circles subtend angles 75° and 120° at their centers, find the ratio of their radii.

Solution:

Here, $75° = 75 \times \pi/180 = 5\pi/12$

And $120° = 120 \times \pi/180 = 2\pi/3$

So, $\theta_1 = 5\pi/12$ and $\theta_2 = 2\pi/3$

Using: $l = r\theta \Rightarrow r_1\theta_1 = r_2\theta_2$

$\Rightarrow r_1/r_2 = (2\pi/3)/(5\pi/12) \Rightarrow r_1/r_2 = 8 : 5$ Ans.

EXAMPLE – 7

The perimeter of a certain sector of a circle is equal to the length of the arc of a semi-circle having the same radius. Express the angle of the sector in degrees, minutes and seconds.

Solution:

Let r be the radius and θ be the angle in radian. Let length of arc = *l*.

Perimeter of a sector = r + r + l = ½ (2πr)

$\Rightarrow 2r + l = \pi r$ ………..(1)

Also, $l/r = \theta$ …………. (2)

By (1) and (2), put $l = r\theta$

$2r + r\theta = \pi r \Rightarrow 2 + \theta = \pi \Rightarrow \theta = \pi - 2$

Angle of sector = $(\pi - 2)$ radian = $(\pi - 2) \times 180°/\pi$

$= (22/7 - 2) \times (180° \times 7)/22$

$= (720/11)° = 65° + (5/11)°$

$= 65° + (5 \times 60)/11 = 65° + 27' + (3/11)'$

$= 65°27' + (3 \times 60)/11 = 65°27'16''$ (App) Ans.

Try yourself:

1. Find the radius of the circle in which a central angle of 60° intercept an arc of length 37.4 cm ($\pi = 22/7$).
   Ans: 35.7
2. A wheel makes 360 revolutions in 1 minute. Through how many radians does it turn in 1 second? Ans: 12π.
3. If sinx = ¾, and x ∈ 2nd Q, then find all the trigonometric ratios.
4. Find the value of: (a) $\tan 15\pi/4$ (b) $\cos(-19\pi/3)$.
   Ans: (a) – 1 (b) ½.
5. In a circle of diameter 40 cm, the length of a chord is 20 cm. Find the length of minor arc of the chord.
6. Find the measure of the angle subtended at the center of a circle of radius 100 cm by an arc of length 22 cm. ($\pi = 22/7$).

EXAMPLE – 8

Prove that:

$2\sec^2\theta - 2\csc^2\theta - \sec^4\theta + \csc^4\theta = \cot^4\theta - \tan^4\theta$.

Solution:

LHS: $2\sec^2\theta - 2\text{cosec}^2\theta - \sec^4\theta + \text{cosec}^4\theta$

$= 2(1 + \tan^2\theta) - 2(1 + \cot^2\theta) - (1 + \tan^2\theta)^2 + (1 + \cot^2\theta)^2$

$= 2 + 2\tan^2\theta - 2 - 2\cot^2\theta - 1 - \tan^4\theta$

$- 2\tan^2\theta + 1 + \cot^4\theta + 2\cot^2\theta$

$= \cot^4\theta - \tan^4\theta$ = RHS. Proved.

EXAMPLE – 9

If $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$, show that: $\cos\theta - \sin\theta = \sqrt{2}\sin\theta$.

Solution:

We have $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta \Rightarrow \cos\theta = \sin\theta/(\sqrt{2} - 1)$

$\Rightarrow \cos\theta = \frac{(\sqrt{2}+1)}{(\sqrt{2}-1)} \sin\theta \Rightarrow \cos\theta = \frac{(\sqrt{2}+1)}{2-1}\sin\theta$

$\Rightarrow \cos\theta = \sqrt{2}\sin\theta + \sin\theta \Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$ Proved.

EXAMPLE – 10

Prove that $\cos\theta + \sec\theta$ can never be equal to 3/2.

Solution:

Let $\cos\theta + \sec\theta = 3/2 \Rightarrow \cos\theta + 1/\cos\theta = 3/2$

$\Rightarrow 2\cos^2\theta - 3\cos\theta + 2 = 0 \Rightarrow \cos\theta = \frac{3 \pm \sqrt{9-16}}{4} = \frac{3 \pm \sqrt{-7}}{4}$

$\Rightarrow \cos\theta = \frac{3 \pm \sqrt{7}\, i}{4}$, this is impossible, because $\cos\theta$ is a real number.

So, $\cos\theta + \sec\theta$ can never be equal to 3/2. **OR**

Using AM > GM $\Rightarrow \cos x + \sec x > 2$

$\Rightarrow \cos x + \sec x$ cannot be equal to 3/2.

EXAMPLE – 11

Find the maximum value of $12\cos\theta - 9\cos^2\theta$.

Solution:

We have: $12\cos\theta - 9\cos^2\theta = -9(\cos^2\theta - \frac{4}{3}\cos\theta)$

$= -9(\cos^2\theta - \frac{4}{3}\cos\theta + \frac{4}{9} - \frac{4}{9})$

$= -9((\cos\theta - 2/3)^2 - 4/9) = -9(\cos\theta - 2/3)^2 + 4 \le 4$

So, maximum value of $12\cos\theta - 9\cos^2\theta = 4$.

EXAMPLE – 12

If $\sec\theta + \tan\theta = p$, then find the value of $\sec\theta$ and $\tan\theta$.

Solution:

We have: $\sec\theta + \tan\theta = p$

$\sec^2\theta - \tan^2\theta = 1 \Rightarrow \sec\theta - \tan\theta = 1/p$

$\Rightarrow 2\sec\theta = p + 1/p \Rightarrow \sec\theta = (p^2 + 1)/p$

And $\tan\theta = \tan\theta/\sec\theta = (p^2 - 1)/(p^2 + 1)$

EXAMPLE – 13

If $\sin\theta = 2\cos\theta$, find the values of $\sin\theta$, $\cot\theta$.

Solution:

$\sin\theta = 2\cos\theta \Rightarrow \tan\theta = 2 \Rightarrow \cot\theta = ½$.

$\text{cosec}\theta = \sqrt{1 + \cot^2\theta} \Rightarrow \sin\theta = \pm 2/(5)$.

EXAMPLE – 14

The area of a certain sector of a circle is 20 sq units. If the radius of the circle be 6 units, find the angle of the sector.

Solution:

Area = 20, radius (r) = 6.

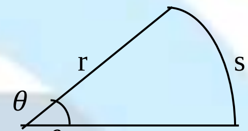
Using the formula:

$A = \frac{1}{2}r^2\theta \Rightarrow \theta = \frac{2A}{r2} = \frac{2 \times 20}{6 \times 6} = \frac{10}{9}\ radians.$

EXAMPLE – 15

The parameter of a certain sector is 10 units and radius of the circle is 3 units, find the area of the sector.

Solution:



We have: $s + 2r = 10$, $s = r\theta$.

So, $r\theta + 2r = 10$. $\theta = \frac{10-2r}{r} = \frac{10-6}{3} = \frac{4}{3}$

Area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 9 \times \frac{4}{3}$ = 6 Sq. Units.

EXAMPLE – 16

Evaluate : (a) $\sin(-1125°)$ (b) $\cos(1410°)$

Solution:

(a) $\sin(-1125°) = -\sin(1125) = -\sin(1080 + 45)$

$= -\sin 45 = \frac{1}{\sqrt{2}}$ Ans.

(b) $\cos(1410°) = \cos(1440 - 30) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$ Ans.

Try yourself:

1. If $\cot x = 3/4$, x lies in IIIrd quadrant. Find all trigonometric ratios.
2. If $\tan x = -5/12$, x lies in II-Q. Find $\sin x$ and $\cos x$.
3. Find the value of $\cot(-15\pi/4)$ and $\sin(-11\pi/3)$.
4. If $\sec A = 5/4$, then find the value of $\frac{4\tan A + 3\cot A - 5\sin A}{5\sin A + 4\sec A - 8\tan A}$.

   Ans: 2

EXAMPLE – 17

Show that: $\frac{\sec 8A - 1}{\sec 4A - 1} = \frac{\tan 8A}{\tan 2A}$.

Solution:

LHS $= \frac{1 - \cos A}{1 - \cos 4A} \times \frac{\cos 4A}{\cos 8A} = \frac{2\sin^2 4A}{2\sin^2 2A} \times \frac{\cos 4A}{\cos 8A} = \frac{\sin 4A \sin 8A}{2\sin^2 2A . \cos 8A}$

$= \tan 8A\left(\frac{\sin 4A}{2\sin^2 2A}\right) = \tan 8A\left(\frac{2\sin 2A\cos 2A}{2\sin^2 2A}\right) = \frac{\tan 8A}{\tan 2A}$ = RHS

EXAMPLE – 18

Find the period of the following functions:

(a) $f(x) = \cos^2 x$ (b) $f(x) = 4\cos x + 3\sin(x/2)$

Solution:

(a) $f(x) = \cos^2 x$. Let T be the period of f(x), then

$f(T + x) = \cos^2(T + x) = \cos^2 x$

By the general formula valid (G. Value)

$x^2 + 2Tx + T^2 \pm x^2 = 2\pi K$

But this is impossible, because K may attain only integer values, while LHS is a quadratic of x. So, $\cos^2 x$ is not a periodic function.

(b) The period of sin $(x/2)$ is 4π and the period of cos x is 2π, so the period of f(x) is 4π. (LCM of periods).

EXAMPLE – 19

Prove that: cos 18 – sin 18 = $\sqrt{2}$ sin27°.

Solution:

LHS = Cos18 – Sin18 = Cos18 – Cos72

$= -2 \text{ Sin} \frac{90}{2} \text{ Sin} \left(\frac{-54}{2}\right) = \sqrt{2} \text{ Sin}27°$. Ans

EXAMPLE – 20

Prove that: $\cos 5A = 16\cos^5 A - 20\cos^3 A + 5\cos A$

Solution:

$\cos 5A = \cos(3A + 2A) = \cos 3A \cos 2A - \sin 3A \sin 2A$

$= (4\cos^3 A - 3\cos A)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)(2\sin A \cos A)$

$= \cos A\,[(4\cos^2 A - 3)(2\cos^2 A - 1) - (3\sin A - 4\sin^3 A)(2\sin A)]$

$= \cos A\,[8\cos^4 A - 10\cos^2 A + 3 + 2(1 - \cos^2 A)(3 - 4)(1 - \cos^2 A)]$

$= \cos A\,[8\cos^4 A - 10\cos^2 A + 3 + 2(1 - \cos^2 A)\times(1 - 4\cos^2 A)]$

$= \cos A\,[8\cos^4 A - 10\cos^2 A + 3 + 2(1 - 5\cos^2 A + 4\cos^4 A)]$

$= 16\cos^5 A - 20\cos^3 A + 5\cos A$

EXAMPLE – 21

Show that: $\cos\frac{\pi}{33}\cos\frac{2\pi}{33}\cos\frac{4\pi}{33}\cos\frac{8\pi}{33}\cos\frac{16\pi}{33} = \frac{1}{32}$.

Solution:

$$\text{LHS} = \frac{\left(2\sin\frac{\pi}{33}\cos\frac{\pi}{33}\right)\left(\cos\frac{2\pi}{33}.\cos\frac{4\pi}{33}.\cos\frac{8\pi}{33}.\cos\frac{16\pi}{33}\right)}{2\sin\frac{\pi}{33}}$$

$$= \frac{\left(2\sin\frac{2\pi}{33}\cos\frac{2\pi}{33}\right)\left(\cos\frac{4\pi}{33}.\cos\frac{8\pi}{33}.\cos\frac{16\pi}{33}\right)}{2.2.\sin\frac{\pi}{33}} \text{ (On simplifying we get)}$$

$$= \frac{\left(2\sin\frac{4\pi}{33}\cos\frac{4\pi}{33}\right)\left(\cos\frac{8\pi}{33}.\cos\frac{16\pi}{33}\right)}{2^3\sin\frac{\pi}{33}} = 1/32.$$

(Using sin2A = 2sinAcosA by same manner)

<u>**Sine Rule:**</u> So, LHS $= \frac{\sin\frac{32\pi}{33}}{2^5\sin\frac{\pi}{33}} = \frac{\sin\left(\pi - \frac{\pi}{33}\right)}{2^5\sin\frac{\pi}{33}} = \frac{1}{2^5} = \frac{1}{32}$ = RHS.

Proved.

EXAMPLE – 22

If A + B = 90°, Find the max value of sinA sinB.

Solution:

Let y = sinA sinB = $\frac{1}{2}$ (2sinA sinB)

$= \frac{1}{2}[\cos(A - B) - \cos(A + B)] = \frac{1}{2}[\cos(A - B) - 0]$

$y = \frac{1}{2}\cos(A - B)$

$y_{max} = \frac{1}{2}$ ($\because$ max cos (A – B) = 1).

EXAMPLE – 23

Find the max and min value of psinA + qcosA.

Solution:

Multiply and divide by $\sqrt{p^2 + q^2}$, we get: Let

T = psinA + qcosA

$= \sqrt{(p^2 + q^2)}\left[\frac{p}{\sqrt{p^2+q^2}}\sin A + \frac{p}{\sqrt{p^2+q^2}}\sin A\right]$

$= \sqrt{(p^2 + q^2)}(\sin A\cos\alpha + \cos A\sin\alpha)$

$= \sqrt{(p^2 + q^2)}\,(\sin(A + \alpha))$

$T_{max} = \sqrt{(p^2 + q^2)}$ ($\because$ $\sin(A + \alpha)_{max} = 1$)

$T_{min} = -\sqrt{p^2 + q^2}$ ($\because$ $\sin(A + \alpha)_{min} = -1$)

EXAMPLE – 24

If mtan(A – 30) = ntan(A + 120), show that: $\cos 2A = \frac{m+n}{2(m-n)}$.

Solution:

We have: $\frac{m}{n} = \frac{\tan(A+120)}{\tan(A-30)}$

Applying Componendo and dividendo rule, we get:

$\frac{m+n}{m-n} = \frac{\tan(A+120)+\tan(A-30)}{\tan(A+120)-\tan(A-30)} = \frac{\sin(A+120+A-30)}{\sin(A+120-A+30)} = \frac{\sin(90+2A)}{\sin 150}$

$= 2\cos 2A \Rightarrow \frac{m+n}{2(m-n)} = \cos 2A$ proved.

EXAMPLE – 25

Show that: (sin3A + sinA)sinA + (cos3A – cosA)cosA = 0.

Solution:

LHS $= \left[2\sin\left(\frac{3A+A}{2}\right)\cos\left(\frac{3A-A}{2}\right)\right]\sin A + \left[2\sin\left(\frac{3A+A}{2}\right)\sin\left(\frac{3A-A}{2}\right)\right]\cos A$

= 2sin2AcosAsinA – 2sin2AsinAcosA = 0 = RHS

EXAMPLE – 26

If x + y + z = xyz, then show that:

$\frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \left(\frac{3x-x^3}{1-3x^2}\right)\left(\frac{3y-y^3}{1-3y^2}\right)\left(\frac{3z-z^3}{1-3z^2}\right)$.

Solution:

Let x = tanA, y = tanB, z = tanC

Then x + y + z = xyz

$\Rightarrow$ tanA + tanB + tanC = tanA tanB tanC

$\Rightarrow A + B + C = \pi$

$\Rightarrow 3A + 3B + 3C = 3\pi n = 3K$ (K = integer)

$\Rightarrow$ tan3A + tan3B + tan3C = tan3A tan3B tan3C

$\Rightarrow \frac{3x-x^3}{1-3x^2} + \frac{3y-y^3}{1-3y^2} + \frac{3z-z^3}{1-3z^2} = \left(\frac{3x-x^3}{1-3x^2}\right)\left(\frac{3y-y^3}{1-3y^2}\right)\left(\frac{3z-z^3}{1-3z^2}\right)$

EXAMPLE – 27

Prove that: sin12.sin48.sin54 = $\frac{1}{8}$.

Solution:

LHS $= \frac{1}{2}$(2sin12.sin48)sin54 $= \frac{1}{2}$(cos36 – cos60).sin54

$= \frac{1}{2}\left(\frac{\sqrt{5}+1}{4} - \frac{1}{2}\right)\left(\frac{\sqrt{5}+1}{4}\right) = \frac{1}{2}\left(\frac{\sqrt{5}-1}{4}\right)\left(\frac{\sqrt{5}+1}{4}\right) = \frac{1}{3^2}(5-1) = \frac{1}{8}$

= RHS.

EXAMPLE – 28

Prove that: tanA + 2tan2A + 4tan4A + 8cot8A = cotA.

Solution:

We have: 2tan2A + 4tan4A + 8cot8A

$= \cot A - \tan A = \frac{1 - tan^2 A}{tanA} = \frac{2}{tan2A} = 2\cot 2A$

$\Rightarrow$ 4tan4A + 8cot8A = 2cot2A – 2tan2A = 4cot4A

$\Rightarrow$ 8cot8A = 4cot4A – 4tan4A = 8cot4A. Which is true.

EXAMPLE – 29

Prove that:

Sin2A + Sin2B + Sin2C = 4SinASinBSinC

Solution:

LHS = Sin2A + 2Sin(B + C). Cos (B – C)

= 2SinACosA + 2SinACos (B – C)

= 2SinA[– Cos (B + C) + Cos (B – C)]

= 2SinA(2 SinBSinC) = 4 SinASinBSinC = RHS.

EXAMPLE – 30

Prove that: $Sin^2 5 + Sin^2 10 + \ldots\ldots + Sin^2 90 = 9\frac{1}{2}$.

Solution:

LHS = $(Sin^2 5 + Sin^2 85) + (Sin^2 10 + Sin^2 80)$

$+ \ldots\ldots + Sin^2 45 + Sin^2 90$

$= (Sin^2 5 + Cos^2 5) + (Sin^2 10 + Cos^2 10)$

$+ \ldots\ldots\ldots + \left(\frac{1}{\sqrt{2}}\right)^2 + 1$

$= (1 + 1 + \ldots\ldots\ldots\ldots + 1) + \frac{1}{2} + 1 = 9 + \frac{1}{2} = 9\frac{1}{2}$ = RHS.

Proved.

EXAMPLE – 31

Find the value of $Cos^4 \frac{\pi}{8} + Cos^4 \frac{3\pi}{8} + Cos^4 \frac{5\pi}{8} + Cos^4 \frac{7\pi}{8}$.

Solution:

We have: $Cos^4 \frac{7\pi}{8} = Cos^4 \frac{\pi}{8}$ And $Cos^4 \frac{5\pi}{8} = Cos^4 \frac{3\pi}{8} = Sin^4 \frac{\pi}{8}$

So, We get: $2(Cos^4 \frac{\pi}{8} + Cos^4 \frac{3\pi}{8})$

$= 2(1 - 2Sin^2 \frac{\pi}{8}. Cos^2 \frac{\pi}{8}) = 2 - (2\,Sin \frac{\pi}{8} Cos \frac{\pi}{8})^2$

$= 2 - Sin^2 \frac{\pi}{4} = 2 - \frac{1}{2} = \frac{3}{2}$ Ans.

EXAMPLE – 32

Find the sum: tanAsec2A + tan2Asec4A + ……… n terms.

Solution:

First term, $t_1 = \frac{SinA}{cosA\,Cos2A}$ = tan2A – tanA

Similarly:

t_2 = tan4A – tan2A

t_3 = tan6A – tan4A

$t_n = tan2^n A - tan2^{n-1} A$

So, $S_n = t_1 + t_2 + t_3 + \ldots\ldots + t_n = tan2^n A - tanA$ Ans.

EXAMPLE – 33

Prove that: $\cos\frac{\pi}{14} + \cos\frac{3\pi}{14} + \cos\frac{5\pi}{14} = \frac{1}{2}\cos\frac{\pi}{14}$.

Solution:

We get: LHS = $\frac{Cos\left(\frac{\pi}{14} + \left(\frac{3-1}{2}\right)\frac{2\pi}{14}\right) Sin\left(\frac{2\pi}{14} \times \frac{3}{2}\right)}{Sin\left(\frac{2\pi}{14} \times \frac{1}{2}\right)} = \frac{2\,Cos\frac{3\pi}{14}.\,Sin\frac{3\pi}{14}}{2Sin\frac{\pi}{14}}$

$= \frac{Sin\frac{6\pi}{14}}{2\,Sin\frac{\pi}{14}} = \frac{Sin\left(\frac{\pi}{2} - \frac{\pi}{14}\right)}{2Sin\frac{\pi}{14}} = \frac{1}{2}\cot\frac{\pi}{14}$ = RHS.

Try yourself:

1. Prove that: cos(π/4 + x) + cos(π/4 – x) = √2.cosx.
2. Prove that: $\frac{sin5x - 2sin3x + sinx}{cos5x - cosx}$ = tanx.
3. Prove that: $\frac{\cos(\pi + x).\cos(-x)}{\sin(\pi - x).\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$.
4. Prove that: $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x.\sin 4x$.
5. Prove that: $\cos^2 2x - \cos^2 6x = \sin 4x.\sin 8x$.
6. Prove that: $\tan 4x = 4\tan x(1 - \tan^2 x)/\{1 - 6\tan^2 x + \tan^4 x\}$.
7. Prove that: $\cos 4x = 1 - 8\sin^2 x.\cos^2 x$.
8. Prove that: $\cos 6x = 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$.
9. Prove that: cotx.cot2x – cot2x.cot3x – cot3x.cotx = 1

TRIGONOMETRICAL EQUATIONS

Some useful formulas

(a) If sinθ = sinα ⇒ θ = nπ + $(-1)^n$α ∀ n∈I.

(b) If cosθ = cosα ⇒ θ = 2nπ ± α ∀ n ∈ I

(c) If tanθ = tanα ⇒ θ = nπ + α ∀ α ∈ I

(d) If $\sin^2\theta = \sin^2\alpha$, $\cos^2\theta = \cos^2\alpha$,

$\tan^2\theta = \tan^2\alpha$ ⇒ θ = nπ ± α. (same for all)

Some useful results

(m) If sinθ = 0 ⇒ θ = nπ, ∀ n ∈ I

(n) If cosθ = 0 ⇒ θ = nπ + π/2, ∀ n ∈ I

(o) If tanθ = 0 ⇒ θ = nπ, ∀ n ∈ I

(p) If sinθ = 1 ⇒ θ = 2nπ + π/2, ∀ n ∈ I

(q) If sinθ = – 1 ⇒ θ = 2nπ – π/2, ∀ n ∈ I

(r) If cosθ = 1 ⇒ θ = 2nπ, ∀ n ∈ I

(s) If cosθ = – 1 ⇒ θ = 2nπ + π, ∀ n ∈ I.

EXAMPLE – 34

Solve the following: $Sin\theta = \frac{\sqrt{3}}{2}$ (b) $\tan\theta = -\sqrt{3}$.

Solution:

(a) If $Sin\theta = Sin\alpha$, then,

G. Solution: $\theta = n\pi + (-1)^n.\alpha$

Now: $Sin\theta = \frac{\sqrt{3}}{2} = Sin\frac{\pi}{3}$

So, G. Solution: $\theta = n\pi + (-1)^n.{}^{\pi}/_{3}$

(b) $\tan\theta = -\sqrt{3} = -tan\,{}^{\pi}/_{3}$

So, G. Solution: $\theta = n\pi + (-{}^{\pi}/_{3})$

$\Rightarrow \theta = n\pi - {}^{\pi}/_{3}$ Ans.

EXAMPLE – 35

Solve the following: -

(a) $Sin^2\theta - 2Cos\theta + \frac{1}{4} = 0$

(b) $\text{Sec}\theta - 1 = (\sqrt{2} - 1)\tan\theta$

Solution:

(a) $\text{Sin}^2\theta - 2\cos\theta + \frac{1}{4} = 0$

On solving we get: $4\text{Cos}^2\theta + 8\cos\theta - 5 = 0$

$\Rightarrow (2\text{Cos}\theta + 5)(2\cos\theta - 1) = 0$

$\Rightarrow \text{Cos}\theta = -{}^{5}/_{2}$ (impossible)

So, $\text{Cos}\theta = {}^{1}/_{2} = \cos{}^{\pi}/_{3}$

So, G. Solution: $\theta = 2n\pi \pm {}^{\pi}/_{3}$ Ans.

(b) $\text{Sec}\theta - 1 = (\sqrt{2} - 1)\tan\theta$

Squaring both the sides –

$(\text{Sec}\theta - 1)^2 = (\sqrt{2} - 1)^2 \tan^2\theta$

$\Rightarrow (\text{Sec}\theta - 1)^2 - (3 - 2\sqrt{2})(\text{Sec}^2\theta - 1) = 0$

$\Rightarrow (\sec\theta - 1)[(\sec\theta - 1) - (3 - 2\sqrt{2})(\sec\theta + 1)] = 0$

$\Rightarrow (\text{Sec}\theta - 1)(\sec\theta - \sqrt{2})(2\sqrt{2} - 2) = 0$

Now, $\text{Sec}\theta = 1 \Rightarrow \cos\theta = 1 = \cos 0$

$\Rightarrow$ G. Solution: $\theta = 2n\pi$ Ans.

If $\text{Sec}\theta - \sqrt{2} = 0 \Rightarrow \text{Sec}\theta = \sqrt{2}$

$\Rightarrow \text{Cos}\theta = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$

So, G. Solution: $\theta = 2n\pi \pm {}^{\pi}/_{4}$ Ans.

EXAMPLE – 36

Solve: $\tan\theta + \tan 2\theta + \sqrt{3}\tan\theta\tan 2\theta = \sqrt{3}$

Solution:

$\tan 2\theta + \tan\theta = \sqrt{3}(1 - \tan\theta\tan 2\theta)$

$\Rightarrow \frac{\tan 2\theta + \tan\theta}{1 - \tan\theta\tan 2\theta} = \sqrt{3} \Rightarrow \tan 3\theta = \sqrt{3} = \tan{}^{\pi}/_{3}$

G. Solution:

$3\theta = n\pi + {}^{\pi}/_{3} \Rightarrow \theta = \frac{\pi}{9}(3n + 1)$ Ans.

EXAMPLE – 37

Solve: $\text{Cos}3\theta + 2\cos\theta = 0$.

Solution:

$\text{Cos}3\theta + 2\cos\theta = 0 \Rightarrow 4\text{Cos}^3\theta - \cos\theta = 0$

$\Rightarrow \text{Cos}\theta\,(4\text{Cos}^2\theta - 1) = 0$

$\Rightarrow \text{Cos}\theta = 0$ OR $\text{Cos}\theta = \pm\frac{1}{2}$

G. Solution:

$\theta = n\pi + {}^{\pi}/_{2}$ OR $\text{Cos}\theta = \frac{1}{2} \Rightarrow \theta = 2m\pi \pm \frac{\pi}{3}$

$\text{Cos}\theta = -\frac{1}{2} \Rightarrow \theta = (2m + 1)\pi + \frac{\pi}{3}$

$\theta = m\pi \pm \frac{\pi}{3}$

EXAMPLE – 38

Solve the following:

(a) $\text{Cot}4\theta = \cot\theta$

(b) $\text{SinP}\theta = \sin q\theta$

(c) $\text{Cos}\, m\theta = \sin n\theta$

Solution:

(a) $\text{Cot}\, 4\theta = \cot\theta$

G. Solution: $\Rightarrow 4\theta = n\pi + \theta \Rightarrow \theta = {}^{n\pi}/_{3}$

(b) $\text{SinP}\theta = \sin q\theta$

G. Solution: $\text{P}\theta = n\pi + (-1)^n.\, q\theta$

When n = even, then putting n = 2m, so:

$P\theta = 2m\pi + q\theta \Rightarrow \theta = \frac{2m\pi}{P - q}$ Ans.

When n = odd, then putting n = 2m + 1, so:

$P\theta = (2m + 1)\pi - q\theta \Rightarrow \theta = \frac{(2m+1)\pi}{p+q}$ Ans.

(c) $\text{Cos}\, m\theta = \sin n\theta \Rightarrow \text{Cos}\, m\theta = \cos(\frac{\pi}{2} - n\theta)$

G. Solution: $\text{m}\theta = 2K\pi \pm (\frac{\pi}{2} - n\theta)$

Taking + ive Sign: $\text{m}\theta = 2K\pi + \frac{\pi}{2} - n\theta$

$\Rightarrow \theta = \frac{\pi}{2(m+n)}(4K + 1)$

Taking –ive Sign: $\text{m}\theta = 2K\pi - \frac{\pi}{2} + n\theta$

$\Rightarrow \theta = \frac{\pi}{2(m+n)}(4K - 1)$ Ans.

EXAMPLE – 39

Solve: $\tan^2 3\theta = \text{Cot}^2\theta$.

Solution:

We have: $\tan^2 3\theta = \cot^2\theta$

$\Rightarrow \tan 3\theta = \pm\cot\theta = \tan(\frac{\pi}{2} \mp \theta)$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{2} \mp \theta$

Taking + ive sign:

$3\theta = n\pi + \frac{\pi}{2} + \theta \Rightarrow \theta = \frac{\pi}{4}(2n + 1)$ Ans.

Taking – ive Sign:

$3\theta = n\pi + \frac{\pi}{2} - \theta \Rightarrow \theta = \frac{\pi}{8}(2n + 1)$ Ans.

EXAMPLE – 40

Solve: $\text{Sin}x + \sin 2x + \sin 3x = 1 + \text{Cos}x + \cos 2x$.

Solution:

$(\sin x + \sin 3x) + \sin 2x$

$= 1 + \cos x + 2\cos^2 x - 1\ 2\sin 2x\cos x + \sin 2x$

$= \cos x(1 + 2\cos x)$

$\Rightarrow \cos x\,(2\cos x + 1)(2\sin x - 1) = 0$

G. Solution:

If $\cos x = 0 \Rightarrow x = 2x\,\pi + \frac{\pi}{2}$ Ans.

If $2\cos x + 1 = 0 \Rightarrow x = 2n\pi \pm \frac{\pi}{6}$ Ans.

EXAMPLE – 41

Solve: $\text{Sin}\theta = \frac{1}{\sqrt{2}}$ and $\text{Sec}\theta = -\sqrt{2}$.

Solution:

If $Sin\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45°, 135°.$

If $Sec\theta = -\sqrt{2} \Rightarrow \theta = 135°, 225°$

Common value of $\theta = 135°$

So, G. Solution: $\theta = 2n\pi + 3\,{}^{\pi}/_{4}$ Ans.

EXAMPLE – 42

Solve: $\sqrt{3}Cos\theta + Sin\theta = \sqrt{2}.$

Solution:

We have: $\sqrt{a^2 + b^2} = 2$

Dividing by 2: $\Rightarrow \frac{\sqrt{3}}{2}\,Cos\theta + \frac{1}{2}\,Sin\theta = \frac{\sqrt{2}}{2}$

$\Rightarrow Cos\left(\theta - \frac{\pi}{6}\right) = Cos\,\frac{\pi}{4}$

G. Solution: $\theta - \frac{\pi}{6} = 2n\pi \pm \frac{\pi}{4}$

$\Rightarrow \theta = 2n\pi \pm \frac{\pi}{4} + \frac{\pi}{6}$ Ans.

Try yourself:

1. Find the value of tan13π/12.
2. Find the value of tanπ/8.
3. Prove that: cos(3π/4 + x) + cos(3π/4 – x) = – $\sqrt{2}$cosx.
4. Prove that: tan5x – tan3x – tan2x = tan5x.tan3x.tan2x.
5. Prove that: $\frac{sinx-sin3x}{(sinx+cosx)(sinx-cosx)}$ = 2sinx.
6. Prove that: cos2x.cosx/2 – cos3x.cos9x/2 = sin5x.sin5x/2.
7. Prove that: $\cos^2 x + \cos^2(x + \pi/3) + \cos^2(x - \pi/3) = 3/2$.
8. Prove that: 2cosπ/13.cos9π/13 +cos3π/13 + cos5π/13 = 0
9. Prove that: sin3x + sin2x – sinx = 4sinx.cosx/2.cos3x/2.
10. Solve the following:

(a) sin2x + cosx = 0

(b) cos3x + cosx – cos2x = 0

(c) cos4x + cosx = 0

(d) $2\cos^2 x + 3\sin x = 0$

(e) $\sec^2 2x = 1 - \tan 2x$

EXAMPLE – 43

If $Sin^4\theta + Cos^4\theta = {}^{7}/_{2}\ Sin\theta \times Cos\theta,$ then solve for θ.

Solution:

$Sin^4\theta + Cos^4\theta = \frac{7}{2}\,Sin\theta\,Cos\theta$

$\Rightarrow 1 - 2Sin^2\theta\,Cos^2\theta = \frac{7}{2}\,Sin\theta\,Cos\theta$

$\Rightarrow 2(2 - Sin^2 2\theta) = 7 \times 2Sin\theta\,Cos\theta$

$\Rightarrow 4 - 2Sin^2\theta = 7Sin2\theta \Rightarrow 2Sin^2 2\theta + 7Sin2\theta - 4 = 0$

$\Rightarrow (2Sin2\theta - 1)(Sin2\theta + 4) = 0$

$\Rightarrow Sin2\theta = \frac{1}{2}$ OR $Sin2\theta = -4$ (impossible)

So, G. Solution: $\theta = \frac{n\pi}{2} + (-1)^n.\frac{\pi}{12}$ Ans.

EXAMPLE – 44

Solve: $Sec4\theta - Sec2\theta - 2 = 0.$

Solution:

$\frac{1}{Cos4\theta} - \frac{1}{Cos2\theta} = 2 \Rightarrow Cos2\theta - Cos4\theta = 2Cos4\theta\,Cos2\theta$

$\Rightarrow Cos2\theta - Cos4\theta = Cos6\theta + Cos2\theta$

$\Rightarrow Cos6\theta + Cos4\theta = 0 \Rightarrow 2Cos5\theta Cos\theta = 0$

If $Cos5\theta = 0 \Rightarrow \theta = \frac{n\pi}{5} + \frac{\pi}{10}$ Ans.

If $Cos\theta = 0 \Rightarrow \theta = n\pi + \frac{\pi}{2}$ Ans.

EXAMPLE – 45

Solve: sin3a = 4sina.sin(x – a).sin(x + a).

Solution:

$Sin3a = 4Sina\,(Sin^2x - Sin^2a)$

$\Rightarrow 3Sina - 4Sina\,Sin^2x = 0$

$\Rightarrow Sina(3 - 4Sin^2x) = 0 \Rightarrow 4Sin^2x - 3 = 0$

$\Rightarrow Sin^2x = (\frac{\sqrt{3}}{2})^2 = Sin^2\frac{\pi}{3}$

So, G. Solution: $x = n\pi \pm \frac{\pi}{3}.$ Ans.

EXAMPLE – 46

Solve for A, if cosA = cosB and sinA = sinB.

Solution:

We have: $CosA - CosB = 0 \Rightarrow 2sin\frac{A+B}{2}.sin\frac{B-A}{2} = 0$

$\Rightarrow 2Sin\frac{A-B}{2}.Sin\frac{A-B}{2} = 0$ And $SinA - SinB = 0$

$\Rightarrow 2Cos\frac{A+B}{2}.Sin\frac{A-B}{2} = 0$

Common form is $\frac{A+B}{2}$, So:

$Sin\frac{(A-B)}{2} = 0 \Rightarrow \frac{A-B}{2} = n\pi \Rightarrow A = 2n\pi + B$ Ans.

EXAMPLE – 47

Solve for x and y :

$x\,Cos^3y + 3x\,Cosy\,Sin^2y = 14$

$x\,Sin^3y + 3x\,Cos^2y\,Siny = 13.$

Solution:

Dividing both the equation: -

$\frac{Cos^3y+3Cosy\,Sin^2y}{Sin^3y+3Cos^2y\,Siny} = \frac{14}{13}$

Applying com – dived, we get

$\left(\frac{Cosy+Siny}{Cosy-Siny}\right)^3 = 27 = 3^3$

$\Rightarrow \frac{Cosy+Siny}{Cosy-Siny} = 3$ OR $\frac{1+\tan y}{1-\tan y} = 3$

Again applying com-dived, we get: tany = 1/2

CASE – I

If tan y lie in I[st] Quadrant, then

$Siny = {}^{1}/_{\sqrt{5}}$ and $Cosy = {}^{2}/_{\sqrt{5}}$

So, $x = 5\sqrt{5}$ (by eg (1))

CASE – II

If tan y lie in 3[rd] Quadrant, then

$Siny = -{}^{1}/_{\sqrt{5}}, Cosy = -{}^{2}/_{\sqrt{5}}$

$\Rightarrow x = -5\sqrt{5}$ (by eg (1))

So, $x = \pm 5\sqrt{5}$ and $y = tan^{-1}\left(\frac{1}{2}\right)$ Ans.

EXAMPLE – 48

Solve for x and y: $2^{Sinx+Cosy} = 1$ and $16^{Sin^2x+Cos^2y} = 4.$

Solution:

$2^{Sinx+Cosy} = 1 = 2° \Rightarrow Sinx + Cosy = 0 \ldots\ldots\ldots (1)$

And $16^{Sin^2x+Cos^2y} = 4 = 16^{1/2}$

$\Rightarrow Sin^2x + Cos^2y = \frac{1}{2}$ ……..(2)

From eq (1) and (2):

$2Sin^2x = \frac{1}{2} \Rightarrow Sinx = \pm\frac{1}{2}$

Now, if: $Sinx = \frac{1}{2} \Rightarrow Cosy = -\frac{1}{2}$

So, G. Solution: $x = n\pi + (-1)^n.{}^{\pi}/_{6}$

And $y = 2n\pi \pm 2{}^{\pi}/_{3}$ Ans.

If $Sinx = -\frac{1}{2} \Rightarrow Cosy = \frac{1}{2}$

So, G. Solution: $x = n\pi - (-1)^n.{}^{\pi}/_{6}$

And $y = 2n\pi \pm {}^{\pi}/_{3}$ Ans.

TRIGONOMETRY PROBLEMS

1. Find the value of the following:
   (a) tan(19π/3) (b) sin(– 11π/3) (c) cos(– 1710^0)
   Ans: (a) $\sqrt{3}$ (b) $\frac{\sqrt{3}}{2}$ (c) 0
2. Find all the trigonometric ratios if tanx = 1, x ∈ III–Q.
   Ans: cotx = 1, sinx = – $1/\sqrt{2}$,
   cosx = – $1/\sqrt{2}$, secx = – $\sqrt{2}$, cosecx = – $\sqrt{2}$
3. Show that: tan3x.tan2x.tanx = tan3x – tan2x – tanx.
4. Prove that: $\frac{\sin 5x - 2\sin 3x + \sin x}{\cos 5x - \cos x}$ = tanx.
5. Prove that : $\frac{\tan(\frac{\pi}{4}+x)}{\tan(\frac{\pi}{4}-x)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$.
6. Prove that : cot4x(sin5x + sin3x) = cotx(sin5x – sin3x).
7. Prove that : $\cos(3\frac{\pi}{4} + x) - \cos(3\frac{\pi}{4} - x) = -\sqrt{2}\sin x$.
8. Prove that: $\sin^2 6x - \sin^2 4x$ = sin2x.sin10x.
9. Prove that : tan4x = $\frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$.
10. Prove that: cos6x = $32\cos^6x - 48\cos^4x + 18\cos^2x - 1$.
11. Solve for x: (a) $2\cos^2x + 3\sin x = 0$ (b) cos4x = cos2x.
    Ans: (a) $x = n\pi + (-1)^n\frac{7\pi}{6}$ (b) $x = n\pi$, $x = \frac{n\pi}{3}$.
12. Find the value of tan(π/8). Ans: $\sqrt{2} - 1$.
13. If tanx = 3/4, x ∈ III-Q, then find the value of sinx/2, tanx/2. Ans: sinx/2 = $3/\sqrt{10}$, tanx/2 = – 3.
14. Prove that:
    $\cos^2x + \cos^2(x + \pi/3) + \cos^2(x - \pi/3) = 3/2$.
15. Prove that : 2cosx(π/13).cos(9π/13)
    + cos(3π/13) + cos(5 π/13).
16. Prove that : sinx + sin3x + sin5x + sin7x
    = 4cosx.cos2x.sin4x.
17. Prove that: sin3x + sin2x – sinx = 4sinx.cosx/2.cos3x/2.
18. Prove that:
    (a) $(\cos x - \cos y)^2 + (\sin x - \sin y)^2 = 4\sin^2(x - y)/2$.
    (b) cos2x.cos(x/2) – cos3x.cos(9x/2) = sin5x.sin(5x/2).
    (c) 2cos(π/13).cos(9π/13) + cos(3π/13) + cos(5π/13) = 0.

OTHER PROBLEMS

1. Find the degree measure of the following:
   (a) – $(1/4)^c$ (b) $(\pi/8)^c$
   Ans: (a) 14^0 19′5′′ (b) 20^0 30′
2. Find the radian measure of the following:
   (a) –37˚ 30′ (b) 520^0
   Ans: (a) 5π/12 (b) 26π/9
3. Find the angle (in degree) subtended to the center of a circle of radius 25 cm by an arc of length 11 cm.
   Ans: 25^0 12′.
4. The angle of a Δ ABC are in AP. The number of degrees in the greatest is to the numbers of radians in the least is π : 60. Find the angle in degrees and radians.
   Ans: 30, 60, 90
5. If the arc of 15 cm length in two circles, subtend angles of 60˚ and 75˚ at their centers. Find the ratio of their radii.
   Ans: 5 : 4.
6. The minute hand of a watch is 1.5 cm long. How far does it tip move in 40 min? (use 3.14). Ans: 6.28 cm
7. Find the angle between the hours hand and the minute hand of a clock when the time is 7 : 20 AM.
   Ans: 220, 120 and difference = 100.
8. Find the magnitude in degree and radians of the interior angle of a regular polygon, which have:
   (a) heptagon (b) octagon.
   Ans: (a) $(5\pi/7)^c$, 128^0 34’ 17’’ (b) $(3\pi/4)^c$, 135
9. The angle of a Δ ABC are in AP, such that the greatest is the 5 times the least. Find the angle in degree and radian.
   Ans: π/9, π/3, 5π/9
10. The radius of a circle is 30 cm. Find the length of an arc of this circle, if the length of the chord of the arc is 30 cm. Ans: 20π/3
11. The number of sides of two regular polygons are as 5 : 4, and the difference between their angle is 9˚. Find the number of sides of the polygons. Ans: 10, 8
12. The angle of a quadrilateral are in AP and the greatest angle is 120˚. Find the angle in radians.
    Ans: π/3, 4π/9, 5 π/9, 2π/3.
13. Prove that :
    (a) $(\sin\theta + \mathrm{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = \tan^2\theta + \cot^2\theta + 7$.
    (b) (tanθ – secθ + 1)/(tanθ + secθ – 1) = cosθ/(1 + sinθ)
    (c) $\cot^4\theta + \cot^2\theta = \mathrm{cosec}^4\theta - \mathrm{cosec}^2\theta$
14. If cosθ + sinθ = √2cosθ, prove that: cosθ – sinθ = √2sinθ.
15. If acosθ + bsinθ = x and asinθ – bcosθ = y

prove that $x^2 + y^2 = a^2 + b^2$.

16. If $\tan x + \sin x = m$, $\tan x - \sin x = n$, show that : $(m - n)(m + n) = 4\sqrt{mn}$.

17. If $p\cos\theta - q\sin\theta = k$, show that: $p\sin\theta + q\cos\theta = \pm\sqrt{(p^2 + q^2 - k^2)}$.

18. $\sin\theta/\sin\varphi = x$, $\cos\theta/\cos\varphi = y$, find $\tan\theta$ and $\tan\varphi$. Ans:(a) $\pm\frac{x}{y}\frac{\sqrt{y^2-1}}{1-x^2}$ (b) $\pm\frac{\sqrt{y^2-1}}{1-x^2}$

19. If $\sec\theta + \tan\theta = p$, find the values of $\sec\theta$, $\sin\theta$ and $\tan\theta$.
Ans: $\sec\theta = (p^2 + 1)/2p$,
$\sin\theta = (p^2 - 1)/(p^2 + 1)$, $\tan\theta = (p^2 - 1)/2p$.

20. Prove that:
(a) $\cos\theta(\tan\theta + 2)(2\tan\theta + 1) = 5\sin\theta + 2\sec\theta$.
(b) $(1 + \tan\theta.\tan\phi)^2 + (\tan\theta - \tan\phi)^2 0020 \sec^2\theta.\sec^2\phi$.

21. If $\tan^2\theta + k^2 = 1$, prove that: $\sec\theta + \text{cosec}\theta.\tan^3\theta = (2 - k^2)^{3/2}$.

22. Find 'x' if
(a) $\sin(90^\circ + A) = x\cos A\cot(90^\circ + A) + \text{cosec}(90^\circ + A)$.
(b) $\text{cosec}(90^\circ+A) + \sin A\tan(90^\circ+A) + x\cot(90^\circ+A) = 0$.
Ans: (a) tanA (b) sinA

23. Prove that: $\cos510.\cos330 + \sin390.\cos120 = -1$.

24. If $\tan A = p/(p + 1)$ and $\tan B = 1/(2p + 1)$, prove that: $A + B = \pi/4$.

25. If (a) $(1 + \tan\theta)(1 + \tan\phi) = 2$,
(b) $(\cot\theta - 1)(\cot\phi - 1) = 2$, prove that: $\theta + \phi = \pi/4$.

26. If $\cos(A + B) = 4/5$, $\sin(A - B) = 5/13$, and $A, B \in (0 , \pi/4)$, prove that: $\tan2A = 56 / 33$.

27. Prove that: $\tan20^\circ + 2\tan50^\circ = \tan70^\circ$.

28. If $\frac{1}{p}\times\tan(A + B) = \tan(A - B)$, show that $(p + 1)\sin2B = (n - 1)\sin2A$.

29. If $\cot(\pi\sin\alpha) = \tan(\pi\cos\alpha)$, prove that: $\cos(\alpha - \pi/4) = 1/(2\sqrt{2})$.

30. If θ and φ are the solutions of the equation $a\tan\delta + b\sec\delta = k$ then show that: $\tan(\theta + \varphi) = 2ak/(a^2 - k^2)$.

31. If θ and φ are the solutions of the equation $p\cos\delta + q\sin\delta = k$ then show that
(a) $\cos(\theta - \varphi) = [2k^2 - (p^2 - q^2)]/\{p^2 + q^2\}$
$= 2k^2/(p^2 + q^2) - 1$.
(b) $\cos(\theta + \varphi) = (p^2 - q^2)/(p^2 - q^2)$

32. If $\cos\theta + \sin\varphi = p$ and $\sin\theta + \cos\varphi = q$ prove that: $2\sin(\theta + \varphi) = p^2 + q^2 - 2$.

33. Prove that: $\tan8\theta - \tan6\theta - \tan2\theta = \tan8\theta.\tan6\theta.\tan2\theta$.

34. Prove that:
(a) $(\sin\theta + \sin\varphi)/(\cos\theta + \cos\varphi) = \tan(\theta + \varphi)/2$.
(b) $(\cos^2\alpha + \cos^2\beta) + (\sin^2\alpha + \sin^2\beta)$
$+ 2(\sin\alpha.\sin\beta + \cos\alpha.\cos\beta) = 4.\cos^2\{(\alpha - \beta)/4\}$.

35. Prove that:
(a) $\frac{\sin8\theta.\cos5\theta + \cos12\theta.\sin9\theta}{\cos8\theta.\cos5\theta - \cos12\theta.\cos9\theta} = \cot4\theta$
(b) $\frac{\cos2\theta + \cos3\theta + \cos4\theta}{\sin2\theta + \sin3\theta + \sin4\theta} = \cot3\theta$.

36. Prove that:
$$\{(\cos\alpha + \cos\beta)/(\sin\alpha - \sin\beta)\}^n + \{(\sin\alpha + \sin\beta)/(\cos\alpha - \cos\beta)\}^n = \begin{cases} 2\cot^n(\alpha - \beta)/2, & \text{n is even.} \\ 0, & \text{n is odd.} \end{cases}$$

37. If $\sin2\theta = k\sin2\varphi$, prove that: $\tan(\theta + \varphi)/\tan(\theta - \varphi) = (k + 1)/(k - 1)$

38. Show that: $\sqrt{2 + \sqrt{2 + \sqrt{2 + 2\cos8\theta}}} = 2\cos\theta$.

39. Prove that: $\tan8\theta/\tan2\theta = (\sec8\theta - 1)/(\sec4\theta - 1)$.

40. Prove that:
$\cos\theta.\cos2\theta.\cos2^2\theta$.-------- $.\cos2^n\theta = \sin2^n\theta/(2^n.\sin\theta)$ and hence find: $\cos\pi/7.\cos2\pi/7.\cos4\pi/7$.

41. Prove that: $\cos\theta.\cos(60^\circ - \theta).\cos(60^\circ + \theta) = \frac{1}{4}\cos3\theta$.

42. Find $\cot(15/2)^\circ$ and $\tan(15/2)^\circ$.

43. If $\tan\theta = -4/3$, θ lie in 2nd quadrant then find $\sin\theta/2$, $\cos\theta/2$.

44. Prove that: (a) $\sin18^\circ = (\sqrt{5} - 1)/4$
(b) $\sin36^\circ = \sqrt{(10 - 2\sqrt{5})}/4$.

45. In Δ ABC, prove that:
(a) $\sin2A + \sin2B + \sin2C = 4\sin A\sin B\sin C$.
(b) $\cos2A + \cos2B - \cos2C = 1 - 4\sin A\sin B\sin C$
(c) $\cos A + \cos B - \cos C = -1 + 4\cos A/2.\cos B/2.\sin C/2$.
(d) $\cos^2A + \cos^2B - \cos^2C = 1 - 2\sin A\sin B\cos C$.

46. Prove that:
(a) $\cos^2A/2 + \cos^2B/2 - \cos^2C/2 = 2\cos A/2.\cos B/2.\sin C/2$.
(b) $\cot A/2 + \cot B/2 + \cot C/2 = \cot A/2\cot B/2\cot C/2$.
(c) $\tan2A + \tan2B + \tan2C = \tan2A.\tan2B.\tan2C$

47. If $x + y + z = xyz$, prove that:
(a) $2x/(1 - x^2) + 2y/(1 - y^2) + 2z/(1 - z^2)$
$= 8xyz/[(1 - x^2)(1 - y^2)(1 - z^2)]$
(b) $(3x - x^3)/(1 - 3x^2) + (3y - y^3)/(1 - 3y^2)$
$+ (3z - z^3)/(1 - 3z^2)$
$= (3x - x^3)/(1 - 3x^2) \times (3y - y^3)/$
$(1 - 3y^2) \times (3z - z^3)/(1 - 3z^2)$

48. If $\sin\theta + \text{cosec}\theta = 2$, prove that: $\sin^2\theta + \text{cosec}^2\theta = 2$.

49. Prove that: $(3 + \cos4\theta)(1 - 2\sin^2\theta) = 4(\cos^8\theta - \sin^8\theta)$.

50. If $x = \sin^2\theta + \cos^4\theta$, prove that: $3/4 \le x \le 1$.

51. Find the value of $\tan9 - \tan27 - \tan63 + \tan81$. Ans: 4

52. Prove that: $\frac{\sec8x-1}{\sec4x-1} = \frac{\tan8x}{\tan2x}$.

53. Find the value of
$(1 + \cos\pi/8)(1 + \cos3\pi/8)(1 + \cos5\pi/8)$
$\times(1 + \cos7\pi/8)$. Ans: 1/8

54. If α and β are the roots of the equation

$a\tan\theta + b\sec\theta = c$, then show that: $\tan(\alpha + \beta) = \frac{2ac}{a^2-c^2}$.

55. Find the greatest value of sinx.cosx. Ans: 1/2

56. Find the value of $\sin 20^o.\sin 40^o.\sin 60^o.\sin 80^o$. Ans: 3/16

57. If sinθ and cosθ are the roots of the equation $ax^2 - bx + c = 0$, then find the relation between a, b and c. Ans: $a^2 - b^2 + 2ac = 0$.

58. If 3tan(x – 15) = tan(x + 15), then find x. Ans: π/4.

59. Find the value of x for which the inequality $2^{\sin x} + 2^{\cos x} \geq 2^{1-\frac{1}{\sqrt{2}}}$ holds. Ans: all x ∈ R

60. If tanx = b/a, then find the value of $\sqrt{\frac{a+b}{a-b}} + \sqrt{\frac{a-b}{a+b}}$.

Ans: $\frac{2\cos x}{\sqrt{\cos 2x}}$

61. Prove that: $\sin 4x = 4\sin x.\cos^3 x - 4\cos x.\sin^3 x$

62. If tan(A + B) = p and tan(A – B) = q, then show that tan2A = (p + q)/(1 – pq).

63. If tanx = (sinA – cosA)/(sinA + cosA), then show that : $\sin A + \cos A = \sqrt{2}\cos x$.

64. If tanx = a/b, then prove that: bcos2x + a sin2x = b.

65. Find the value of k if k = sin(π/18)sin(5π/18)sin(7π/18). Ans: 1/8.

66. If sinx + cosx = a, then find the value of |sinx – cosx|. Ans: $\sqrt{2-a^2}$.

67. In a triangle ABC with angle C = 90°, then find equation whose roots are tanA and tanB. Ans: $x^2 - \frac{2}{\sin 2x}x + 1 = 0$.

RELATION BETWEEN SIDES AND ANGLES OF A TRIANGLE

In a Δ ABC let AB = c, BC = a and CA = b. semi-perimeter of Δ ABC, s = (a + b + c)/2 and area of Δ ABC is denoted by S or Δ.

Some rules related to the Δ's

(a) ***Sine rule***

sin A/a = sin B/b = sin C/c (= k or 1/k we may use)

(b) ***Cosine rule***

$\cos A = (b^2 + c^2 - a^2)/2bc$,

$\cos B = (a^2 + c^2 - b^2)/2ac$,

$\cos C = (a^2 + b^2 - c^2)/2ab$.

(c) ***Nepier'sformulae***

(i) $\tan(A - B)/2 = \frac{a-b}{a+b}\cot C/2$.

(ii) $\tan(B - C)/2 = \frac{b-c}{b+c}\cot A/2$

(iii) $\tan(C - A)/2 = \frac{c-a}{c+a}\cot B/2$

(d) ***Projection rule***

(i) bcosC + ccosB = a

(ii) ccosA + acosC = b

(iii) acosB + bcosA = c

(e) ***Formulae of half angles***

(i) sinA/2 = √{(s – b)(s – c)/bc}

(ii) sinB/2 = √{(s – a)(s – c)/ac}

(iii) sinC/2 = √{(s – a)(s – b)/ab}

(iv) cosA/2 = √{s(s – a)/bc}

(v) cosB/2 = √{s(s – b)/ac}

(vi) cosC/2 = √{s(s – c)/ab}

(vii) tanA/2 = √{(s – b)(s – c)/s(s – a)}

(viii) tanB/2 = √{(s – a)(s – c)/s(s – b)}

(ix) tanC/2 = √{(s – a)(s – b)/s(s – c)}

Area of triangle

Area of triangle = ½ × base × height

or Area Δ = √s(s – a)(s – b)(s – c)

And Δ = r × s where r = radius of in-circle

Or Δ = abc/4R where R is the radius of circum-circle

Or Δ = ½ × absinC

Δ = ½ × bcsinA = ½ ×ca sin B

Determination of sinA, sinB, sinC

sinA = 2√s(s – a)(s – b)(s – c)/bc ,

sinB = 2√s(s – a)(s – b)(s – c)/ca

sinC = 2√s(s – a)(s – b)(s – c)/ab

Or sinA = 2Δ/bc , sinB = 2Δ/ca, sinC = 2Δ/ab

$\tan A = \sin A/\cos A = 4\Delta/(b^2 + c^2 - a^2)$,

$\tan B = \sin B/\cos B = 4\Delta/(a^2 + c^2 - b^2)$,

$\tan C = 4\Delta/(a^2 + b^2 - c^2)$.

EXAMPLE – 49

If ∠A = 30°, ∠B = 45°, then find a : c.

Solution:

We have $\frac{\sin A}{a} = \frac{\sin C}{c}$ (sine rule)

So, a : c = sinA: sinC = $\frac{1}{2} : \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right) = \sqrt{2} : \left(\sqrt{3}+1\right)$Answer.

EXAMPLE – 50

If the ration of three angle $\angle A : \angle B : \angle C = 1:2:3$, then find the ratio of their sides.

Solution:

We have $\angle A : \angle B : \angle C = 1:2:3$, then

$\angle A = 30, \angle B = 60, \angle C = 90$. By sine rule:

So, a:b:c = sinA:sinB:sinC = $\sin 30^0 : \sin 60^0 : \sin 90^0$

$= \frac{1}{2} : \frac{\sqrt{3}}{2} : 1 = 1 : \sqrt{3} : 2$ Ans.

EXAMPLE – 51

Prove that: (b – c)cosA/2 = a $\sin\left(\frac{B-C}{2}\right)$.

Solution:

$\left(\frac{b-c}{a}\right) = \frac{k\sin B - k\sin C}{k\sin A}$, (sine rule)

$= \frac{\sin B - \sin C}{\sin A} = \frac{2\cos\frac{B+C}{2}.\sin\frac{B-C}{2}}{2\sin\frac{A}{2}.\cos\frac{A}{2}} = \frac{\sin\frac{B-C}{2}}{\cos\frac{A}{2}}$

$\Rightarrow (b - c)\cos A/2 = a\sin\left(\frac{B-C}{2}\right)$.

EXAMPLE – 52

If $a = \sqrt{37}$, b = 3 and c = 4, find cosA.

Solution:

We have $\cos A = \frac{b^2+c^2-a^2}{2bc}$

$= \frac{9+16-37}{24} = -\frac{1}{2} \Rightarrow \cos A = -\frac{1}{2}$ Ans.

EXAMPLE – 53

If a = 4,b = 5, and c = 8, prove that the greatest angle is 120^0.

Solution:

The greatest side is c = 8

So, $\cos C = \frac{b^2+c^2-a^2}{2ab} = \frac{16+25-81}{40} = -\frac{23}{40} < -\frac{1}{2}$

So, $\cos C < \cos 120^0$ Proved.

EXAMPLE – 54

If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$, then prove that: $\angle C = 60^0$.

Solution:

We have: $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c} \Rightarrow \frac{(b+c)+(a+c)}{(a+c)(b+c)} = \frac{3}{a+b+c}$

$\Rightarrow (a + b + 2c)(a + b + c) = 3(a + c)(b + c)$

$\Rightarrow a^2 + b^2 = ab + c^2 \Rightarrow \frac{a^2+b^2-c^2}{2ab} = \frac{1}{2}$

$\Rightarrow \cos C = \cos 60^0 \Rightarrow C = 60^0$ Answer.

EXAMPLE – 55

Prove that: $\frac{a^2\sin(B-C)}{\sin B+\sin C} + \frac{b^2\sin(C-A)}{\sin B+\sin C} + \frac{c^2\sin(A-B)}{\sin A+\sin B} = 0$.

Solution:

Taking Ist part, we have

$\frac{a^2\sin(B-C)}{\sin B+\sin C} = \frac{a^2(\sin B\cos C-\cos B\sin C)}{bK+cK}$

$= \frac{a^2}{K(b+c)}\left[bK\left(\frac{a^2+b^2-c^2}{2ab}\right) - cK\left(\frac{c^2+a^2-b^2}{2ca}\right)\right]$

$= \frac{a^2K}{2aK(b+c)}[a^2 + b^2 - c^2 - c^2 - a^2 + b^2] = \frac{aK}{2K(b+c)}[2(b^2 - c^2)]$

$= a(b - c) = ab - ac$

Similarly:

IInd part = b (c – a) = bc – ab

And IIIrd part = c(a – b) = (a – bc)

So, LHS = ab – ac + bc – ab +ca –bc = 0. RHS

EXAMPLE – 56

If a = 16, b = 24, c = 20 then find the value of cosA/2 and tanB/2.

Solution: We have: $s = \frac{a+b+c}{2} = 30$

So, $\cos\frac{B}{2} = \sqrt{\frac{s(s-b)}{ac}} = \sqrt{\frac{30(30-24)}{20\times16}} = \sqrt{\frac{30\times6}{20\times16}} = \sqrt{\frac{9}{16}} = \frac{3}{4}$.

$\sin\frac{B}{2} = \sqrt{1-\cos^2 B/2} = \sqrt{1-\frac{9}{16}} = \frac{\sqrt{7}}{4}$

$\Rightarrow \tan\frac{B}{2} = \frac{\sin B/2}{\cos B/2} = \frac{\sqrt{7}}{3}$. Ans.

EXAMPLE – 57

In Δ ABC, prove that: $(b + c + a)\left[\cot\frac{B}{2} + \cot\frac{C}{2}\right] = 2a\cot\frac{A}{2}$.

Solution:

LHS: $(b + c - a)\left[\sqrt{\frac{s(s-b)}{(s-c)(s-a)}} + \sqrt{\frac{s(s-c)}{(s-a)(s-b)}}\right]$

$= 2(s - a)\left[\frac{\sqrt{s(s-b)^2}+\sqrt{s(s-b)^2}}{\sqrt{(s-a)(s-b)(s-c)}}\right] = 2\sqrt{s}(s-a)\left[\frac{2s-b-c}{(s-a)(s-b)(s-c)}\right]$

$= 2\frac{\sqrt{s(s-a)^2}\times a}{\sqrt{(s-a)(s-b)(s-c)}} = 2\sqrt{\frac{s(s-a)}{(s-b)(s-c)}} = 2a\cot A/2$.

EXAMPLE – 58

In ΔABC, prove that:

(b + c)cosA + (c + a)cosB + (a + b)cosC = a + b + c.

Solution:

LHS

bcosA + ccosA + ccosB + acosB +acosC + bcosC

= (ccosA + acosB) + (ccosA + acosC) + (bcosC + ccosB)

= c + b + a = a + b+ c = RHS

EXAMPLE – 59

In, ΔABC, prove that:

$a^3\cos(B - C) + b^3\cos(C - A) + c^3\cos(A - B) = 3abc$

Solution:

Ist Part $= a^3\cos(B - C) = a^2.a\cos(B - C)$

$= ka^2\sin(B + C).\cos(B - C)$

$= \frac{ka^2}{2}[2\sin(B + C)\cos(B - C)]$

$= \frac{ka^2}{2}[\sin 2B + \sin 2C]$

$= \frac{ka^2}{2}[2\sin B\cos B + 2\sin C\cos C]$

$= ka^2\left[\frac{b}{k}\cos B + \frac{c}{k}\cos C\right] = a^2(b\cos B + c\cos C)$

Similarly: -

$II^{nd} = b^2(c\cos C + a\cos A)$

$III^{rd} = c^2(a\cos A + b\cos B)$

On adding:

$LHS = a^2b\cos B + a^2c\cos C + b^2c\cos C$

$+ b^2a\cos A + c^2a\cos A + c^2a\cos C$

$= ab(a\cos B + b\cos A) + ac(a\cos C + c\cos A)$

$+ bc(c\cos C + c\cos B)$

= ab(c) + ac(b) + bc(a) = abc + abc + abc = 3abc = RHS.

EXAMPLE – 60

In ΔABC, $\angle C = 90^0$, then prove that: $\tan\frac{A-B}{2} = \frac{a-b}{a+b}$.

Solution:

We know that: $\Rightarrow \tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot C/2$

$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot 45^0 \Rightarrow \tan\frac{A-B}{2} = \frac{a-b}{a+b}$.

EXAMPLE – 61

If in ΔABC, b + c = 3a, then prove that: $\cot\frac{B}{2}.\cot\frac{C}{2} = 2$.

Solution:

We have b + c = 3a $\Rightarrow$ sinB + sinC = 3sinA (since rule)

$\Rightarrow 2\sin\frac{B+C}{2}.\cos\frac{B-C}{2} = 6\sin\frac{A}{2}\cos\frac{A}{2}$

$\Rightarrow \cos\frac{B-C}{2} = 3\cos\frac{B+C}{2}$

$\Rightarrow \cos\frac{B}{2}\cos\frac{C}{2} + \sin\frac{B}{2}\sin\frac{C}{2} = 3\left\{\cos\frac{B}{2}\cos\frac{C}{2} - \sin\frac{B}{2}\sin\frac{C}{2}\right\}$

$\Rightarrow 2\cos\frac{B}{2}\cos\frac{C}{2} = 4\sin\frac{B}{2}\sin\frac{C}{2} \Rightarrow \cot\frac{B}{2}.\cot\frac{C}{2} = 2$ Proved.

EXAMPLE – 62

If cosA + 2cosB + cosC = 2, prove that a, b, c, are in A.P.

Solution:

cosA + cosC = 2(1 – cosB)

$\Rightarrow 2\cos\frac{A+C}{2}.\cos\frac{A-C}{2} = 2.2\sin^2\frac{B}{2}$

$\Rightarrow 2\cos\frac{A-C}{2} = 2.2\sin\frac{B}{2} \Rightarrow 2\cos\frac{B}{2}\cos\frac{A-C}{2} = 4\sin\frac{B}{2}\cos\frac{B}{2}$

$\Rightarrow$ sinA + sinC = 2sinB $\Rightarrow$ a + c = 2b (sine rule)

Hence a, b, c are in A.P. Proved.

EXAMPLE – 63

If tan A/2 = 5/6, tan B/2 = $\frac{20}{37}$ prove that, a, b, and c are in A.P.

Solution:

We have: $\tan\frac{C}{2} = \cot\frac{B+A}{2} = \frac{\cot A/2.\cot B/2-1}{\cot A/2+\cot B/2}$

$\Rightarrow \tan\frac{C}{2} = \frac{\frac{6}{5}.\frac{37}{20}-1}{\frac{6}{5}+\frac{37}{20}} = \frac{2}{5}$

and $\tan\frac{A}{2}.\tan\frac{C}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} \times \sqrt{\frac{(s-a)(s-b)}{s(s-c)}}$

$\Rightarrow \frac{5}{6}.\frac{2}{5} = \frac{s-b}{s} \Rightarrow$ 3s – 3b = s $\Rightarrow$ a + b + c = 3b $\Rightarrow$ a + c = 2b.

So, a, b, c are in A.P. proved.

EXAMPLE – 64

If cosA + cosB + cosC = 3/2, prove that the ΔABC in equilateral Δ.

Solution:

cosA + cosB + cosC = 3/2

$\Rightarrow \frac{b^2+c^2-a^2}{2bc} + \frac{a^2+c^2-b^2}{2ac} + \frac{a^2+b^2-c^2}{2ab} = \frac{3}{2}$

$\Rightarrow a(b^2+c^2-a^2)+b(a^2+c^2-b^2)+c(a^2+b^2-c^2) = 3abc$

$\Rightarrow a(b^2+c^2) + b(c^2+a^2) + c(a^2+b^2) = a^3+b^3+c^3-3abc$

$\Rightarrow a(b-c)^2 + b(c-a)^2 + c(a-b)^2 = a^3+b^3+c^3-3abc$

$\Rightarrow a(b-c)^2 + b(c-a)^2 + c(a-b)^2 - \frac{1}{2}(a+b+c)[(b-c)^2+(c-a)^2+(a-b)^2] = 0$

$\Rightarrow (b-c)^2(b+c-a) + (c-a)^2(c+a-b) + (a-b)^2(a+b-c) = 0$

Now using: b – c = c – a = a – b = 0

$\Rightarrow$ a = b = c, so ΔABC is a equilateral Δ.

EXAMPLE – 65

If φ is the mid-point of BC, and AO is perpendicular to AC, then show that: cosA.cosC = $\frac{2(c^2-a^2)}{3ac}$.

Solution:

In ΔABC: $\cos C = \frac{a^2+b^2-c^2}{2ab}$…………..(1)

In ΔABC: $\cos A = \frac{b}{\left(\frac{a}{2}\right)} \Rightarrow$ cosA = 2b/a…….(2)

From eq. (1) and eq. (2): $\frac{2b}{a} = \frac{a^2+b^2-c^2}{2ab} \Rightarrow b^2 = \frac{a^2-c^2}{3}$

Now, LHS = cosA.cosC= $\left(\frac{a^2+b^2-c^2}{2ab}\right)\left(\frac{b}{a/2}\right)$

$= \frac{\frac{a^2-c^2}{3}+c^2-a^2}{ac} = \frac{2(c^2-a^2)}{3ac}$. Proved.

EXAMPLE – 66

If $\angle B = 30^0$, $\angle C = 45^0$, inclined side is $(\sqrt{3}+1)$, prove that the area of Δ = $\left(\frac{\sqrt{3}+1}{2}\right)$.

Solution:

$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c} \Rightarrow \frac{\sin 105}{\sqrt{3}+1} = \frac{\sin 30}{b} = \frac{\sin 45}{c}$

$\Rightarrow b = \frac{\sqrt{3}+1}{2\sin 105}, c = \frac{\sqrt{3}+1}{\sqrt{2}\sin 105}$

Area = $\frac{1}{2}bc\sin A = \frac{(\sqrt{3}+1)^2}{2+2\sqrt{2}\sin 105}$

So, Area = $\frac{1}{2}(\sqrt{3}+1)$ Proved.

EXAMPLE – 67

The sides of a Δ are 3, consecutive natural numbers and its longest angle is twice the smallest angle. Prove that the sides of the Δ are 4, 5, 6.

Solution:

$\angle A = 2\angle B$ and $\angle C = 180 - 3\angle B$

By using sine rule: $\frac{\sin A}{n+2} = \frac{\sin B}{n} = \frac{\sin C}{n+1}$

$\underset{\text{I}}{\frac{\sin A}{n+2}} = \underset{\text{II}}{\frac{\sin B}{n}} = \underset{\text{III}}{\frac{\sin C}{n+1}}$

Taking I&II, we get:

cosB = $\frac{n+2}{2n}$ and by I and III we get:

$\frac{\sin 2B}{(n+2)} = \frac{3\sin B-4\sin^3 B}{(n+1)}$

(Putting: A = 2B and C = 180 – 3B)

$\Rightarrow \frac{2\cos B}{n+2} = \frac{3-4\sin^2 B}{n+1} \Rightarrow \frac{2\cos B}{n+2} = \frac{4\cos^2 B-1}{n+1}$

Putting cosB = $\frac{n+2}{2n}$, we get:

$\frac{1}{n} = \frac{\left(\frac{n+2}{n}\right)^2-1}{n+1}$, on solving, we get: n = 4.

Hence sides are 4, 5, 6 proved.

EXAMPLE – 68

In a ΔABC, $c^4 - 2(a^2+b^2)c^2 + a^4 + a^2b^2 + b^4 = 0$, prove that: $\angle C = 60^0$ or 120^0.

Solution:

$c^4 - 2(a^2+b^2)c^2 + a^4 + a^2b^2 + b^4 = 0$

$\Rightarrow a^4 + b^4 + c^2 - 2a^2b^2 - 2b^2c^2 - a^2b^2 + 2a^2b^2 = 0$

$\Rightarrow (a^4 + b^4 - c^2) = (ab)^2 \Rightarrow a^2 + b^2 - c^2 = \pm ab$

$\Rightarrow \frac{a^2+b^2+c^2}{2ab} = \pm\frac{1}{2} \Rightarrow \cos C = \pm\frac{1}{2}$

$\Rightarrow \angle C = 60^0$ or 120^0. Proved.

PROBLEMS BASED ON SINE AND COSINE RULES

1. The angles of a triangle are in AP and it is given that b : c = √3 : √ 2, find the angle A.
2. In any triangle ABC show that:
   (a) asinA – bsinB = csin(A – B).
   (b) a(bcosC – ccosB) = $b^2 - c^2$.
3. If in a triangle ABC, $\frac{sinA}{sinC} = \frac{\sin(A-B)}{\sin(B-C)}$, show that: a^2, b^2, c^2 are in AP.
4. In any triangle show that: $\frac{asecA+bsecB}{tanA+tanB} = \frac{bsecB+csecC}{tanB+tanC} = \frac{csecC+asecA}{tanC+tanA}$.
5. If in triangle ABC, $\frac{cosA}{a} = \frac{cosB}{b}$, prove that the triangle is isosceles.
6. The sides of a triangle are a, b and $\sqrt{a^2+b^2+ab}$, find the greatest angle.
7. In triangle ABC, a = 3, b = 5 and c = 7, find cosC.
8. In a triangle ABC; a, b, c are in AP. Show That: 2sinA/2.sinC/2 = sinB/2.
9. In any triangle ABC show that: $2(a\sin^2\frac{C}{2} + c\sin^2\frac{A}{2}) = c + a - b$.
10. In any triangle ABC if c + a = 2b, prove that: cotA/2.cotC/2 = 3.

Ans: 1. 75^0 6. 120^0 7. – ½.

LOGARITHMS AND SOLUTION OF TRIANGLES

10 + log sinθ = ∠ sinθ

10 + log cosθ = ∠ cosθ

10 + log tanθ = ∠ tanθ

Note:

(a) If θ increases then sinθ, tanθ, secθ increase.

(b) If θ increases, then cosθ, cotθ and cosecθ decrease.

EXAMPLE – 69

If $\sin 43^0 23` = 0.6865761$ and $\sin 43^0 24` = 0.6870875$, then find the value of $\sin 43^0 23`17``$.

Solution:

sin 43^0 24` = 0.6870875

sin43^0 23`= 0.6865761

Difference for 1` = 0.0002114

So, diff for 60`` = 0.0002114

$\Rightarrow$ diff for 17`` = $\frac{0.0002114 \times 17}{60}$ = .0000599

So, sin$43^0$23`17``= 0.6868761 + 0.0000599

= 0.6869360 Ans.

EXAMPLE – 70

If cos32^0 16` = 0.845526 and cos$32^0$ 17`= 0.8454172, then find the value of cos32^0 16` 31``.

Solution:

cos32^0 16` = 0.845526

cos 32^0 17`= 0.8454172

Diff for 1` = 0.0001554

Diff for 60`` = 0.0001554

Diff for 31`` = $\frac{.0001554 \times 31}{60}$ = 0.0000803

So, cos$32^0$16`31``= 0.8455726 – 0.0000803= 0.8454923.

EXAMPLE – 71

If ∠ tan 79^0 51` 41`` = 10.747565 and difference for 1` = 7290, find the value of θ for which value of ∠ tanθ = 10.7475657, ∠ tan $79^0$ 51` 48`` = 10.7476532.

Solution:

Difference = 10.747632 – 10.7475657 = 0.000875

Now: Diff of .0000875 = $\frac{60 \times .0000875}{0.0007290}$ = 7``

So, θ = 79^0 51` 40`` + 7`` **or** θ = 79^0 51` 49`` Answer.

EXAMPLE – 72

If $\cos\frac{c}{2} = \sqrt{\frac{3}{5}}$, then prove that: ∠cos $\frac{c}{2}$ = 9.8890756.

log6 = 0.7781513.

Solution :

$\cos\frac{c}{2} = \left(\frac{6}{10}\right)^{1/2}$. Taking log :

$\log\cos\frac{c}{2} = \frac{1}{2}(\log 6 - \log 10)$ $\quad L\cos\frac{c}{2} = 10 + \frac{1}{2}(\log 6 - 1)$

$L\cos\frac{c}{2} = 9.8890756$ Ans.

EXAMPLE – 73

If any ΔABC, a = 384, b = 330 and c = 90^0, find the remaining angles: log2 = 0.3010300, log11 = 1.0413927, Ltan$49^0$19` = 10.656886 and Ltan$49^0$20` = 10.659441.

Solution:

tanA = $\frac{a}{b} = \frac{128}{110}$ taking log: log tan A = log128 – log110

Ltan A = 10.0658173

Now, Ltan $49^0$20` = 10.0659441

Ltan $49^0$19` = 10.0656886

Diff. for 1` = 0.0002555

Now Ltan A = 10.0658173

Ltan 4919` = 10.0656886

Difference = 0.0001287

Difference for 0.0001287 = $\frac{6 \times 0.0001287}{0.0002555}$ = 30.2``

So, ∠A = $49^0 19`$ 30.2``

∠B = 180 – (∠A + ∠C) = $40^0 40`$29.8``.

EXAMPLE – 74

If any ΔABC, a = 24.2, b = 18.8, c = 27, find the greatest angle. Given: log2 = .3010300, log3 = 0.4771213, log7 = 0.0845098 ∠tan $38^0 20`$ = 9.8980104 and diff for 1` = 2597.

Solution:

$s = \frac{a+b+c}{2} = 35$

C is the greatest side so, ∠C will be greatest

Now tan = $\frac{c}{2} = \sqrt{\frac{(s-a)(s-b)}{s(s-c)}} = \left(\frac{27\times81}{35\times100}\right)$

Taking log:

$10 + \log\tan\frac{C}{2} = \frac{1}{2}[\log 27 + \log 81 - \log 35 - \log 100]$ + 10

$\angle\tan\frac{C}{2} = 9.8978905$

Now ∠tan$38^0 20`$ = 9.8980104

∠tan $\frac{C}{2}$ = 9.8978905

Difference = 0.0001199

Difference for 0.0001199 = $\frac{60\times0.0001199}{0.0002597}$ = 27``

So, $\angle\frac{C}{2}$ = $38^0 19`33``$ ⇒ ∠C = $76^0 39`6``$

EXAMPLE – 75

The angle between two sides 14 and 11 is 60^0. Find all the angles. Given: log2 = 3.010300, log3 = 0.4771213, Ltan$11^0 44`29.5``$ = 9.3177419.

Solution:

$\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$, Taking log:

$10 + \log\tan\frac{B-C}{2} = 10 + \frac{1}{2}\log 3 + 2\log 2 - \log 100$

⇒ Ltan $\frac{B-C}{2}$ = 9.3177419,

Ltan $\frac{B-C}{2}$ = L tan 11^0 44`29.5``⇒ $\frac{B-C}{2}$ = 44`29.5``

⇒ B – C = $23^0 28`59``$ and B + C = 120º

∠B = $71^0 44`29.5``$, ∠C = $48^0 15`30.5``$.

EXAMPLE – 76

If a ΔABC, ∠A = 46^0, ∠B = 75^0 and ∠C = 60^0, prove that: $a + \sqrt{2}c = 26$.

Solution:

$\sin\frac{45}{a} = \sin\frac{75}{6} = \sin\frac{60}{c} \Rightarrow \frac{a}{1/\sqrt{2}} = \frac{b}{(\sqrt{3}+1)/2\sqrt{2}} = \frac{c}{\sqrt{3}/2}$

$\Rightarrow a\sqrt{2} = \frac{2\sqrt{2}b}{\sqrt{3}+1} = \frac{2c}{\sqrt{3}} = k$ (Says)

So, $a = \frac{k}{\sqrt{2}}$, $b = \frac{(\sqrt{3}+1)k}{2\sqrt{2}}$ and $c = \frac{\sqrt{3}k}{2}$

Now LHS: $a + c\sqrt{2} = \frac{k}{\sqrt{2}} + \sqrt{2}\,\frac{\sqrt{3}k}{2} = 2b$.

EXAMPLE – 77

If in any ΔABC, a, c and A are given and two values of side b are b_1, b_2 such that $b_2 = 2b_1$ then prove that:

$3a = c\sqrt{1 + 8\sin^2 A}$.

Solution:

$\cos A = \frac{b^2+c^2-a^2}{2bc} \Rightarrow b^2 - (2c\cos A)\, b + (c^2 - a^2) = 0$

⇒ $b_1 + b_2 = 2c\cos A$ …………….(1)

And $b_1 b_2 = c^2 - a^2$ …………….(2)

But $b_2 = 2b_1$ …………………(3)

From (1) and (3) $3b_1 = 2c\cos A$ …….(4)

From (2) and (3):

$2b_1^2 = c^2 - a^2$………………...(5)

From (4) and (5): $2\left[\frac{2c\cos A}{3}\right]^2 = c^2 - a^2 \Rightarrow \frac{8c^2\cos^2 A}{9} = c^2 - a^2$

$\Rightarrow -8c^2\cos^2 A + 9c^2 = 9a^2 \Rightarrow 9a^2 = -8c^2 + 8c^2(\sin^2 A) + 9c^2$

$= c^2 + 8c^2\sin^2 A \Rightarrow 3a = c\sqrt{1 + 8\sin^2 A}$.

EXAMPLE – 78

In a ΔABC, a = 28, b = 37, ∠A = 35^0 15`, then prove that in the condition of longest (uncertainly) also find all the angles. Given: log 28 = 1.4471580, log 37 = 1.5682017

∠sin 35^0 15` = 9.7612851, ∠sin $49^0$.41`56``= 9.8823288

Solution:

$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b\sin A}{a} = \frac{37}{28}\sin 35^0\ 15`$

Taking log:

10 + log sin B = 10 + log37 – log28 + logsin 35^0 15`

∠ sin B = 9.8823288 ⇒ ∠ B = 49^0 41` 56``

⇒ a < b So, 2 triangles are possible hence, let ∠ B, = 49^0 41` ⇒ ∠ B_2 = 180 – ∠ B_1

∠B_2 = 130^0 18` 4`` Ans.

∠C_1 = 180 – (∠A + ∠B_1)

∠C_1 = 95^0 3` 4`` Ans.

∠C_2 = 180 – (∠A + ∠B_1) or ∠C_2 = 14^0 26` 56`` Ans.

EXAMPLE – 79

If a, b and c are given, then the prove that:

$C_1^2 + C_2^2 - 2C_1C_2\cos^2 A = 4a^2\cos^2 A$.

Solution:

$\cos A = \frac{b^2+c^2-a^2}{2b^2} \Rightarrow c^2 - 2bc\cos A + (b^2 - a^2) = 0$

$C_1^2 + C_2^2 =$ 2bc cosA and $C_1^2 . C_2^2 = b^2 - a^2$

L.H.S.: $C_1^2 + C_2^2 - 2C_1C_2\cos^2 A$

$= (C_1^2 + C_2^2 + 2C_1C_2) - 2C_1C_2(1+\cos 2A)$

$= (C_1 + C_2)^2 - 2C_1C_2(1 + \cos A)$

$= 4b^2\cos^2 A - 2(b^2 - a^2)2\cos^2 A = 4a^2\cos^2 A$ = RHS.

<u>Some other formulas</u>

r = Δ/s = (s – a)tan(A/2), r_1 = Δ/(s – a) = stan(A/2) ,

r_2 = Δ/(s – b) = s tan (B/2), r_3 = Δ/(s – c) = s tan (C/2) **Or**

r = 4R sin A/2 sin B/2 sin C/2

$r_1 = 4R \sin A/2 \cos B/2 \cos C/2$,
$r_2 = 4R \sin B/2 \cos A/2 \cos C/2$,
$r_3 = 4R \sin C/2 \cos A/2 \cos B/2$

NOTE-

(a) $r_1 + r_2 + r_3 = 4R + r$, $r_1 r_2 + r_2 r_3 + r_3 r_1 = r_1 r_2 r_3/r = s^2$
and $R = a/2 \sin A = b/2 \sin B = c/2 \sin C$

(b) Ptolemy's theorem-
In a cyclic quadrilateral, sum of the product of opposite sides = The product of the diagonals

(c) Centroid, circumcentre and orthocentre are always collinear.

(d) **m – n theorem**;
(i) $m \cot \alpha - n \cot \beta = (m + n) \cot \theta$
(ii) $n \cot B - m \cot C = (m + n) \cot \theta$

Distance of orthocentre from verticess

AN = 2RcosA, CN = 2RcosC, BN = 2RcosB

Distance from sides

DN = 2RcosBcosC, EN = 2RcosAcosB
FN = 2RcosAcosC

Distance of circumcircle from sides

OD = R cos C, OE = R cos A, OF = R cos B

About a regular polygon

n = number of sides, a = length of side, r = radius of incircle,
R = radius of circumcircle.
So, sum of all angles = $(n - 2) \times 180°$
Measurement of one interior angle = $[(n - 2) \times 180°]/n$
Measurement of one exterior angle
= 180° – measurement of one interior angle.
$r = a/\{2\tan(\pi/n)\}$, $R = a/\{2\sin(\pi/n)\}$

EXAMPLE – 80

Find the area of Δ, where sides are 25, 60 and 65.

Solution:

We have: $s = \frac{a+b+c}{2} = 75$

Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{75(75-25)(75-60)(75-65)} = 750$ sq. unit.

EXAMPLE – 81

In a triangle ABC, a = 12, b = 14 and ∠c = 30^0, find the area of the Δ.

Solution:

Area $= \frac{1}{2} ab \sin C = \frac{1}{2} \times 12 \times 14 \times \sin 30^0$

= 42 sq. unit.   Ans.

EXAMPLE – 82

In a triangle ABC, a = 8cm, ∠B = 30^0, ∠C = 60^0, find the area of triangle.

Solution:

Area $= \frac{a^2 \sin B \sin C}{2\sin(B+C)} = \frac{8\times 8\times \sin 30^0 \sin 60^0}{\sin 90^0} = 8\sqrt{3}$ sq. unit.

EXAMPLE – 83

Prove that: acosA + bcosB + ccosC = 4R sinA sinB sinC.

Solution:

Putting a = 2RsinA, b = 2RsinB, c = 2RsinC
LHS:
2RsinAcosA + 2RsinBcosB + 2RsinccosC)
= R(sin2A + sin2B + sin2C) = 4R sinA sinB sinC = RHS.

EXMAPLE – 84

Prove that : $b^2\sin 2C + c^2 \sin 2B = 4\Delta$.

Solution:

LHS : $2b^2 \sin C \cos C + 2c^2 \sin B \cos B$

$= 2b^2 \times \frac{c}{2R}\left(\frac{a^2+b^2-c^2}{2ab}\right) + \frac{2c^2.b}{2R}\left(\frac{a^2+c^2-b^2}{2ac}\right)$

$= \frac{bc}{2aR} \times 2a^2 = \frac{abc}{R} = 4\Delta$ = RHS

EXAMPLE – 85

If a = 13, b = 14, c = 15, find r.

Solution:

$s = \frac{a+b+c}{2} = 21$, $\Delta = \sqrt{s(s-a)(s-b)(s-c)} = 84$

$r = \frac{\Delta}{s} = \frac{84}{21} = 4$.   Ans.

EXAMPLE – 86

Prove that: 4Rr cosA/2 cos B/2 cos C/2 = Δ.

Solution:

LHS: $4 \times \frac{abc}{4\Delta} \times \frac{\Delta}{s} \times \sqrt{\frac{s(s-a)}{bc}} \times \sqrt{\frac{s(s-b)}{ac}} \times \sqrt{\frac{s(s-c)}{ab}}$

$= \sqrt{s(s-a)(s-b)(s-c)} = \Delta$.   Proved.

EXAMPLE – 87

Prove that : 4R.rs = abc

Solution:

LHS: $4 \times \frac{abc}{4\Delta} \times \frac{\Delta}{s} \times s = abc$ = RHS. Proved.

EXAMPLE – 88

In a triangle ABC, a = 15, b = 20 and c = 25. Find in-radius, ex-radius and radius of circumcircle.

Solution:

$s = \frac{a+b+c}{2} = 30$

$\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$= \sqrt{30(30-15)(30-20)(30-25)}$ = 150 sq. unit

$R = \frac{abc}{4\Delta} = 12.5$ sq. unit, $r = \frac{\Delta}{s} = 5$ unit,

$r_1 = \frac{\Delta}{s-a} = 10$ unit, $r_2 = \frac{\Delta}{s-b} = 15$ unit, $r_3 = \frac{\Delta}{s-c} = 30$ unit.

EXAMPLE – 89

Prove that: $\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{1}{r}$.

Solution:

LHS $= \frac{s-a}{\Delta} + \frac{s-b}{\Delta} + \frac{s-c}{\Delta} = \frac{s}{\Delta} = \frac{1}{r}$ = RHS.

EXAMPLE – 90

In a triangle ABC, r_1, r_2 and r_3 are in H.P.. Prove that a, b, and c are in A.P.

Solution:

r_1, r_2 and r_3 are in H.P. so, $r_2 = \frac{r_1 r_3}{r_1+r_3} \Rightarrow \frac{\Delta}{s-b} = \frac{2\frac{\Delta}{(s-a)}\cdot\frac{\Delta}{(s-c)}}{2\frac{\Delta}{(s-a)}+2\frac{\Delta}{(s-c)}}$

$\Rightarrow \frac{1}{(s-b)} = \frac{2}{2s-c-a} \Rightarrow \frac{2}{2s-2b} = \frac{2}{b} \Rightarrow b = 2s - 2b \Rightarrow 2b = a + c$

Hence a, b, and c in A.P. proved.

EXAMPLE – 91

Prove that: $r_1 + r_2 + r_3 - r = 4r$.

Solution:

LHS = $r_1 + r_2 + r_3 - r$

$= \Delta\left[\frac{1}{s-a}+\frac{1}{s-b}+\frac{1}{s-c}-\frac{1}{s}\right] = \Delta\left[\frac{s-b+s-a}{(s-a)(s-b)}+\frac{s-s+c}{s(s-c)}\right]$

$= \Delta\left[\frac{2s-(a+b)}{(s-a)(s-b)}+\frac{c}{s(s-c)}\right] = \Delta\, c\left[\frac{s(s-c)+(s-a)(s-b)}{s(s-a)(s-b)(s-c)}\right]$

$= \frac{\Delta c}{\Delta^2}[2s^2 - s(a+b+c) + ab]$

$= \frac{c}{\Delta}[ab] = \frac{abc}{\Delta} = 4R = \text{RHS}.$

EXAMPLE – 92

Prove that: $(r_1 - r)(r_2 - r)(r_3 - r) = 4Rr^2$.

Solution:

LHS: $(r_1 - r)(r_2 - r)(r_3 - r)$

$= \Delta^3\left(\frac{1}{s-a}-\frac{1}{s}\right)\left(\frac{1}{s-b}-\frac{1}{s}\right)\left(\frac{1}{s-c}-\frac{1}{s}\right)$

$= \Delta^3\left(\frac{a}{s(s-a)}\right)\left(\frac{b}{s(s-b)}\right)\left(\frac{c}{s(s-c)}\right) = \frac{\Delta^3 abc}{s^2\Delta^2} = \frac{\Delta abc}{s^2}$

$= 4r^2R$ = RHS.

EXAMPLE – 93

In a triangle ABC, $\angle C = 90^0$, Prove that: $R + r = \frac{1}{2}(a+b)$.

Solution:

$2R = \frac{c}{\sin C} = \frac{c}{\sin 90} = c$

$r = (s-c)\tan\frac{C}{2} = (s-c)$

Now: $2r = 2s - 2c = a + b - c \Rightarrow 2r = a + b - 2R$

$\Rightarrow 2(r + R) = a + b \Rightarrow (R + r) = \frac{1}{2}(a+b)$. Proved.

EXAMPLE – 94

If cotA, cotB and cotC are in A.P. then prove that a^2, b^2 and c^2 are in A.P.

Solution:

cotA, cotB and cotC are in A.P. so,

cotA – cotB = cotB – cotC

$\frac{\cos A}{\sin A} - \frac{\cos B}{\sin B} = \frac{\cos B}{\sin B} - \frac{\cos C}{\sin C}$

$\frac{\text{Sin}(B-A)}{\text{Sin}A} = \frac{\text{Sin}(c-b)}{\text{Sin}c}$

Sin(A+B) sin(B–A)= Sin(B+C) Sin(C–B)

$\sin^2 b - \sin^2 A = \sin^2 C - \sin^2 B$

$\frac{b^2}{4R^2} - \frac{a^2}{4R^2} = \frac{c^2}{4R^2} - \frac{b^2}{4R^2} \Rightarrow b^2 - a^2 = c^2 - b^2$

$2b^2 = a^2 + c^2$. Hence a^2, b^2, c^2 are in A.P.

EXAMPLE – 95

If p, q and r respectively the perpendicular from circumcenter to the sides of the ΔABC, prove that: $\frac{a}{p}+\frac{b}{q}+\frac{c}{r} = \frac{abc}{4pqr}$.

Solution:

We have: P = RcosA, q = RcosB, r = RcosC

LHS: $\frac{a}{2R\cos A}+\frac{b}{2R\cos B}+\frac{c}{2R\cos C}$

$= \frac{2R\sin A}{R\cos A}+\frac{2R\sin B}{R\cos B}+\frac{2R\sin C}{R\cos C}$

= 2(tanA + tanB + tanC) = 2(tanA tanB tanC)

$= 2\left[\frac{\sin A}{\cos A}\frac{\sin B}{\cos B}\frac{\sin C}{\cos C}\right] = \frac{2}{8R^3}\left[\frac{abc}{\cos A\cos B\cos C}\right] = \frac{abc}{4pqr}$ = RHS.

EXAMPLE – 96

In a triangle ABC, ∠A, ∠B, ∠C are in A.P. prove that: $2\cos\frac{A-C}{2} = \frac{a+c}{\sqrt{a^2+c^2-ac}}$.

Solution:

∠A, ∠B, ∠C are in A.P. , so, $\angle B = 60^0$

$\frac{a^2+b^2+c^2}{2ac} = 60^0 = \frac{1}{2} \Rightarrow a^2 - ac + c^2 = b^2$ ….(1)

Now, RHS: $\frac{a+c}{\sqrt{a^2+c^2-ac}} = \frac{a+c}{b}$

$= \frac{k(\sin A+\sin C)}{k\sin B} = \frac{\sin A+\sin c}{\sin B} = 2\cos\frac{A-C}{2}$ = LHS.

Try yourself:

1. If the sides of a triangle are 13, 14 and 15, then find: (a) Δ (b) r (c) R.
2. In any triangle ABC; show that:
   (a) sinA + sinB + sinC = s/R.
   (b) sinA.sinB.sinC = $\Delta/2R^2$.
3. In any triangle ABC; show that:
   $(b + c)\sec\frac{(B-C)}{2} = 4R\cos\frac{A}{2}$.

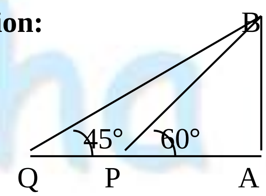
Answers: 1. (a) 84, (b) 4, (c) 8.125

HEIGHT & DISTANCE

EXAMPLE – 97

A boat is being rowed away from a cliff 300m high. At the top of the cliff the angle of depression of the boat changes from 60° to 45° in 2 minutes. Find the speed of the boat.

Solution:



Let AB = 300m,

PA = 300 Cot 60° = $300/\sqrt{3}$m

QA = 300 Cot 45° = 300m

$PQ = 300 - \frac{300}{\sqrt{3}} = 300 - 100\sqrt{3}$

PQ = $100(3 - \sqrt{3})$m Ans.

Speed = $\frac{1}{2} \times 100\ (3 - \sqrt{3}) \times 60 = 3000\ (3 - \sqrt{3})\ {}^{m}/_{h}$.

EXAMPLE – 98

A boy standing 30m south of a pole of height K, walks 60m towards East and find the angle of elevation of the top of the tower to be 30°. Find the volume of K.

Solution:

By the figure: (Draw the figure)

$OB = K\ Cot\ 30° = K\sqrt{3}$

In $\triangle$ ABO: $OB^2 = OA^2 + AB^2$

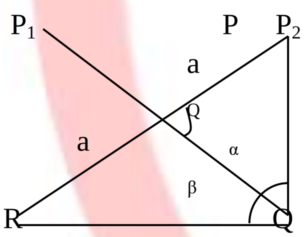
$3K^2 = (30)^2 + (60)^2 = 4500 \Rightarrow K = 10\sqrt{15}m$.

EXAMPLE – 99

A Bus travelling on one of two intersecting roads, subtends at a certain angles on the other road, an angle α, when the front of the bus reaches the junction and an angle β, when the end of the bus reaches it. Prove that the two roads are inclined to each other at an angle θ, determined by $\theta = Cot^{-1}(\frac{Cot\ \alpha - Cot\ \beta}{2})$.

Solution:

Let P_1, and P_2 are two roads.



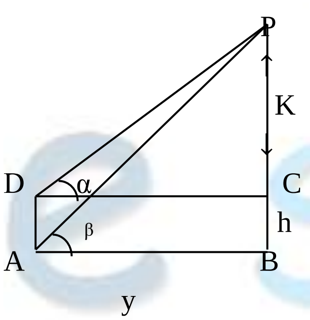
In $\triangle$PQR, By m – n theorem: $(a + a)Cot\ \theta = aCot\alpha - aCot\beta$

$\Rightarrow \theta = Cot^{-1}(\frac{Cot\alpha - Cot\beta}{2})$ Proved.

EXAMPLE – 100

The top of a hill observed from the top and bottom of a building of height h is at angles of elevation $\alpha\ and \beta$ respectively. Prove that the height of the hill is $(\frac{h\ Cot\ \alpha}{Cot\ \alpha - Cot\ \beta})$.

Solution:



AD = Building and BP = Hill

$tan\beta = \frac{h+x}{y}$ and $tan\ \alpha = \frac{x}{y}$

Eliminating 'y', we get: $tan\beta = \frac{h+x}{xCot\alpha}$

$\Rightarrow x = \frac{h\ Cot\beta}{Cot\alpha - Cot\beta} \Rightarrow x + h = \frac{h\ Cot\alpha}{Cot\alpha - Cot\beta}$ Proved.

SOME OBJECTIVE EXAMPLES

EXAMPLE – 1

If $\pi < 2\theta < 3\pi/2$, then $\sqrt{2 + \sqrt{2 + 2cos2\theta}}$ is equal to

(a) –2cosθ/2 (b) –2sinθ/2

(c) 2cosθ/2 (d) 2sinθ/2

Solution:

$\sqrt{2 + \sqrt{2(1 + cos2\theta)}} = \sqrt{2 + \sqrt{4cos^2\theta}} = \sqrt{2(1 + cos\theta)}$

$= 2cos\theta/2$

Hence (c) is correct answer.

EXAMPLE – 2

If tanθ = √n for some non-square natural number n, then sec2θ is

(a) a rational number (b) an irrational number

(c) a positive number (d) none

Solution:

$sec2\theta = (1 + tan^2\theta)/(1 - tan^2\theta) = (1 + n)/(1 - n)$

Where n is a non-negative natural number so that $1 - n \neq 0$.

$\Rightarrow$ sec2θ is a negative rational number.

Hence (a) is correct answer.

EXAMPLE – 3

The minimum value of cos(cosx) is

(a) 0 (b) –cos1 (c) cos1 (d) –1

Solution:

cosx varies from – 1 to 1 for all real x.

thus cos(cosx) varies from cos1 to cos0.

$\Rightarrow$ minimum value of cos(cos1) is cos1.

Hence (c) is correct answer.

EXAMPLE – 4

If $sinx = cos^2x$, then $cos^2x(1 + cos^2x)$ is

(a) 0 (b) 1 (c) 2 (d) none

Solution:

We have $cos^2x(1 + cos^2x) = cos^2x + cos^4x$

$= cos^2 x + sin^2x = 1$.

Hence (b) is correct answer.

EXAMPLE – 5

The maximum value of $4sin^2x + 3cos^2x + sinx/2 + cosx/2$ is

(a) 4 + √2 (b) 3 + √2 (c) 9 (d) 4

Solution:

Maximum value of $4sin^2x + 3cos^2x$ is $sin^2x + 3 = 4$.

And that of $sinx/2 + cosx/2 = \sqrt{2}$, at $x = \pi/2$.

Hence maximum value of function is $4 + \sqrt{2}$.

Hence (a) is correct answer.

EXAMPLE – 6

If sinα and sinβ and cosα are in GP, then roots of the equation $x^2 + 2x\cot\beta + 1 = 0$ are always

(a) equal (b) real

(c) imaginary (d) greater than 1

Solution:

Sinα, sinβ and cosα are in GP.

$\Rightarrow \sin^2\beta = \sin\alpha\cos\alpha \Rightarrow \cos2\beta = 1 - \sin2\alpha \geq 0$.

Now the discriminant of the given equation is

$4\cot^2\beta - 4 = 4\cos2\beta.\mathrm{cosec}^2\beta \geq 0 \Rightarrow$ real roots.

Hence (b) is correct answer.

EXAMPLE – 7

If $S = \cos^2\pi/n + \cos^2 2\pi/n + ... + \cos^2(n-1)\pi/n$,

then S equals to

(a) n(n+1)/2 (b) (n–1)/2 (c) (n–2)/2 (d) n/2

Solution:

$S = \cos^2\pi/n + \cos^2 2\pi/n + ... + \cos^2(n-1)\pi/n$

$= \frac{1}{2}\{1 + \cos2\pi/n + 1 + \cos4\pi/n + 1$

$+ \cos6\pi/n + \ldots + 1 + \cos2(n-1)\pi/n\}$

$= \frac{1}{2}\{n - 1 + \sum_{k=1}^{n-1}\cos\left(\frac{2k\pi}{n}\right)\} = (n-2)/2$.

Hence (c) is correct answer.

EXAMPLE – 8

If in a triangle ABC,

$\sin^2A + \sin^2B + \sin^2C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled

(c) acute angled (d) obtuse angled

Solution:

We have $\sin^2A + \sin^2B + \sin^2C = 2 \Rightarrow 2\cos A.\cos B.\cos C = 0$

$\Rightarrow$ either $A = 90^0$ Or $b = 90^0$ Or $c = 90^0$.

Hence (b) is correct answer.

EXAMPLE – 9

If sinθ is GM of sinϕ and cosϕ, then the value of cos2θ is

(a) $\cos^2(\pi/4 + \phi)$ (b) $2\cos^2(\pi/4 + \phi)$ (c) $3\cos^2(\pi/4 + \phi)$ (d) none

Solution:

$2\sin^2\theta = 2\sin\phi\cos\phi \Rightarrow 1 - \cos2\phi = \sin2\phi$

$\Rightarrow \cos2\theta = 1 - \sin2\phi = 1 + \cos(\pi/2 + 2\phi)$

$\Rightarrow \cos2\theta = 2\cos^2(\pi/4 + \phi)$.

Hence (b) is correct answer.

EXAMPLE – 10

If $4n\alpha = \pi$, then $\cot\alpha.\cot2\alpha.\cot3\alpha.......\cot(2n-1)\alpha$ is equal to

(a) 0 (b) 1 (c) n (d) none

Solution:

Given $4n\alpha = \pi \Rightarrow 2n\alpha = \pi/2$

Now: $\cot\alpha.\cot(2n-1)\alpha = \cot\alpha.\cot(\pi/2 - \alpha) = \cot\alpha.\tan\alpha = 1$.

Similarly, $\cot2\alpha.\cot(2n-2)\alpha = 1$,

$\cot3\alpha.\cot(2n-3)\alpha = 1$…….

So, $\cot\alpha.\cot2\alpha.\cot3\alpha.......\cot(2n-1)\alpha = 1.1.1.1 = 1$.

Hence (b) is correct answer.

EXAMPLE – 11

If $\cos5\theta = a\cos\theta + b\cos^3\theta + c\cos^5\theta + d$, then

(a) a = 20 (b) b = – 30

(c) a + b + c = 2 (d) a + b + c + d = 1

Solution:

$\cos5\theta = \cos(2\theta + 3\theta) = \cos2\theta.\cos3\theta - \sin2\theta.\sin3\theta$

$= (2\cos^2\theta - 1)(4\cos^3\theta - 3\cos\theta)$

$- (2\sin\theta\cos\theta)(3\sin\theta - 4\sin^3\theta) \Rightarrow \cos5\theta =$

$16\cos^5\theta - 20\cos^3\theta + 5\cos\theta$.

Hence (d) is correct answer.

EXAMPLE – 12

If in a triangle ABC,

(sinA + sinB + sinC)(sinA + sinB – sinC)

= 3sinAsinB, then angle C is equal to

(a) π/6 (b) π/4 (c) π/3 (d) 5π/12

Solution:

We have:

(sinA + sinB + sinC)(sinA + sinB + sinC) = 3sinA.sinB

$\Rightarrow (\sin A + \sin B)^2 - \sin^2C = 3\sin A.\sin B$

$\Rightarrow \sin^2A + \sin^2B - \sin^2C = \sin A.\sin B$

$\Rightarrow \sin^2A + \sin(B + C).\sin(B - C) = \sin A.\sin B$

$\Rightarrow \sin A[\sin(B + C) + \sin(B - C)] = \sin A.\sin B$

$\Rightarrow \sin A(2\sin B.\sin C) = \sin A.\sin B \Rightarrow \cos C = ½ \Rightarrow C = \pi/3$.

Hence (c) is correct answer.

EXAMPLE – 13

If tan A – tan B = x and cot B – cot A = y, then cot (A–B) is equal to

(a) $\frac{1}{y} - \frac{1}{x}$ (b) $\frac{1}{x} - \frac{1}{y}$ (c) $\frac{1}{x} + \frac{1}{y}$ (d) none

Solution:

We have,

tan A – tan B = x and cot B – cot A = y

Now, $\cot B - \cot A = y \Rightarrow \frac{\tan A - \tan B}{\tan A \tan B} = y \Rightarrow \frac{x}{\tan A \tan B} = y$

$\Rightarrow \tan A \tan B = \frac{x}{y}$

Now, $\cot(A-B) = \frac{1}{\tan(A-B)} = \frac{1 + \tan A \tan B}{\tan A \ \tan B} = \frac{1+\frac{x}{y}}{x} = \frac{x+y}{xy} = \frac{1}{x} + \frac{1}{y}$

Hence (a) is correct answer.

EXAMPLE – 14

If sin α + sin β = a and cos α + cos β = b then sin(α + β) is equal to

(a) $\frac{2ab}{a^2-b^2}$ (b) $\frac{ab}{a^2-b^2}$ (c) $\frac{ab}{a^2+b^2}$ (d) $\frac{2ab}{a^2+b^2}$

Solution:

Now, $\sin(\alpha+\beta) = \sqrt{1-\cos^2(\alpha+\beta)}$

$= \sqrt{1-\left(\frac{b^2-a^2}{b^2+a^2}\right)^2}$, using above example

$= \sqrt{\frac{4a^2b^2}{(a^2+b^2)^2}} = \frac{2ab}{b^2+a^2}$

(or divide 1 by 2 and solve)

Hence (d) is correct answer.

EXAMPLE – 15

If sinθ = nsin(θ + 2α), then tan (θ + α) is equal to

(a) $\frac{1+n}{2-n}$ tanα (b) $\frac{1-n}{1+n}$ tanα (c) tanα (d) $\frac{1+n}{1-n}$ tanα

Solution:

We have, sinθ = n sin (θ+ 2α)

$\Rightarrow \frac{\sin(\theta+2\alpha)}{\sin\theta} = \frac{1}{n} \Rightarrow \frac{\sin(\theta+2\alpha)+\sin\theta}{\sin(\theta+2\alpha)-\sin\theta} = \frac{1+n}{1-n}$

$\Rightarrow \frac{2\sin(\theta+\alpha)\cos\alpha}{2\sin\alpha\cos(\theta+\alpha)} = \frac{1+n}{1-n} \Rightarrow \tan(\theta+\alpha) = \frac{1+n}{1-n}\tan\alpha$

Hence (d) is correct answer.

EXAMPLE – 16

If $\tan^2\theta = 2\tan^2\phi + 1$, then $\cos 2\theta + \sin^2\phi$ is equal to

(a) 1 (b) 2 (c) –1 (d) none

Solution:

We have, $\cos 2\theta = \frac{1-\tan^{2\theta}}{1+\tan^2\theta} = \frac{1-(2\tan^2\phi+1)}{1+2\tan^2\phi}$

[so, $\tan^2\theta = 2\tan^2\phi + 1$]

$= \frac{-2\tan^2}{2+2\tan^2\phi} = \frac{-\tan^2}{\sec^2\phi} = -\sin^2\phi$ so, $\cos 2\theta + \sin^2\phi = 0$

Hence (d) is correct answer.

EXAMPLE – 17

If $\theta = \frac{\pi}{2^n+1}$, then $\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta n$ is equal to

(a) $\frac{1}{2^n}$ (b) cos θ (c) 2 (d) 2^n

Solution:

We have,

$\cos\theta \cos 2\theta \cos 2^2\theta \ldots \cos 2^{n-1}\theta = \left\{\frac{\sin 2^n\theta}{2^n \sin\theta}\right\} = \frac{\sin 2^n\theta}{2^n\sin\theta}$

$= \frac{\sin(\pi-\theta)}{2^n \sin\theta}$ [Given, $\theta = \frac{\pi}{2^n+1} \Rightarrow 2^n\theta + \theta = \pi$]

$= \frac{1}{2^n}$ $\Rightarrow 2^n\theta = \pi - \theta$

Hence (a) is correct answer.

EXAMPLE – 18

The maximum and minimum values of 7cosθ + 24 sin θ are equal to

(a) 25 and – 25 (b) 24 and – 24
(c) 5 and – 5 (d) none

Solution:

We know that the maximum and minimum values of acosθ + bsinθ are $\sqrt{a^2+b^2}$ and $-\sqrt{a^2+b^2}$ respectively.

Hence, the maximum and minimum values of 7cosθ + 24sinθ are $\sqrt{7^2+24^2}$ and $-\sqrt{7^2+24^2}$ are equals to + 25 and – 25 respectively.

Hence (a) is correct answer.

EXAMPLE – 19

The minimum value of cos(cosx) is

(a) 0 (b) – cos1 (c) cos1 (d) – 1

Solution:

cosx varies from –1 to 1 for all real n.

Thus cos(cosx) varies from cos1 to cos 0

⇒ minimum value of cos(cosx) is cos 1

Hence (c) is correct answer.

EXAMPLE – 20

If $\sin x = \cos^2 x$, then $\cos^2 x\,(1+\cos^2 x)$ equals to

(a) 0 (b) 1 (c) 2 (d) none

Solution:

$\cos^2 x(1+\cos^2 x) = \cos^2 x + \cos^4 x = \cos^2 x + \sin^2 x = 1$

Hence (b) is correct answer.

EXAMPLE – 21

The maximum value of $4\sin^2 x + 3\cos^2 x + \sin\frac{x}{2} + \cos\frac{x}{2}$ is

(a) $4+\sqrt{2}$ (b) $3+\sqrt{2}$ (c) 9 (d) 4

Solution:

Maximum value of $4\sin^2 x + 3\cos^2 x$ i.e. $\sin^2 x + 3$ is 4 and that of $\sin\frac{x}{2} + \cos\frac{x}{2}$ is $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}$ both attained at $x = \frac{\pi}{4}$.

Hence the given function has maximum value $4+\sqrt{2}$.

Hence (a) is correct answer.

EXAMPLE – 22

If in a Δ ABC, ∠C = 90^0, then the maximum value of sin A sin B is

(a) $\frac{1}{2}$ (b) 1 (c) 2 (d) none

Solution:

sinA.sinB = $\frac{1}{2}$ × 2 sinA sinB = $\frac{1}{2}$[cos(A – B) – cos(A + B)]

= $\frac{1}{2}$[cos(A – B) – cos 90^0] = $\frac{1}{2}$ cos(A – B) ≤ $\frac{1}{2}$

⇒ maximum value of sinA.sinB = $\frac{1}{2}$

Hence (a) is correct answer.

EXAMPLE – 23

The maximum value of sin(cosx) is equal to

(a) sin1 (b) 1 (c) $\sin\left(\frac{1}{\sqrt{2}}\right)$ (d) $\sin\left(\frac{\sqrt{3}}{2}\right)$

Solution:

cosx ∈ [–1, 1] ∀ x ∈ R and sin x is increasing in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

⇒ maximum value of sin(cosx) = sin1.

Hence (a) is correct answer.

EXAMPLE – 24

If α + β + γ = π, then the value of

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is equal to

(a) 2sin α (b) 2sinα cosβ sinγ

(c) 2sinα sinβ cos γ (d) 2sinα sinβ sinγ

Solution:

$\sin^2\alpha + \sin(\beta - \gamma)\sin(\beta + \gamma)$

$= \sin^2\alpha + \sin(\pi - \alpha)\sin(\beta - \gamma)$

$= \sin\alpha\,[\sin\alpha + \sin(\beta - \gamma)]$

$= \sin\alpha\,[\sin(\beta - \gamma) + \sin(\beta + \gamma)] = 2\sin\alpha\sin\beta\cos\gamma$

Hence (c) is correct answer.

EXAMPLE – 25

In a ΔABC, if cotA cotB cotC > 0, then the triangle is

(a) acute angled (b) right angled

(c) obtuse angled (d) does not exist

Solution:

Since cotA cotB cotC > 0

cotA cotB cotC are positive ⇒ triangle is acute angled.

Hence (a) is correct answer.

EXAMPLE – 26

If sinα, sin β and cos α are in G.P., then roots of the equation $x^2 + 2x\cot\beta + 1 = 0$ are always

(a) equal (b) real

(c) imaginary (d) greater than 1

Solution:

sin α , sin β and cos α are in G.P ⇒ $\sin^2\beta = \sin\alpha\cos\alpha$

$\Rightarrow \cos 2\beta = 1 - \sin 2\alpha \geq 0$

Now, the discriminant of the given equation is

$4\cot^2\beta - 4 \geq 0$ ⇒ Roots are always real.

Hence (b) is correct answer.

EXAMPLE – 27

If $S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \ldots\ldots + \cos^2\frac{(n-1)\pi}{n}$ then S equals to

(a) $\frac{n}{2}(n+1)$ (b) $\frac{1}{2}(n-1)$ (c) $\frac{1}{2}(n-2)$ (d) $\frac{n}{2}$

Solution:

$S = \cos^2\frac{\pi}{n} + \cos^2\frac{2\pi}{n} + \ldots + \cos^2(n-1)\frac{\pi}{n}$

$= \frac{1}{2}\left[1 + \cos\frac{2\pi}{n} + 1 + \cos\frac{4\pi}{n} + 1 + \cos\frac{6\pi}{n} + \ldots + 1 + \cos 2(n-1)\frac{\pi}{n}\right]$

$= \frac{1}{2}\left[n - 1 + \sum_{k=1}^{n-1}\cos\frac{2k\pi}{n}\right] = \frac{1}{2}[n - 1 - 1] = \frac{1}{2}(n-2)$

Hence (c) is correct answer.

EXAMPLE – 28

If in a Δ ABC, $\sin^2A + \sin^2B + \sin^2C = 2$, then the triangle is always

(a) isosceles triangle (b) right angled

(c) acute angled (d) obtuse angled.

Solution:

$\sin^2A + \sin^2B + \sin^2C \Rightarrow 2\cos A.\cos B.\cos C = 0$

⇒ either $A = 90^0$ or $B = 90^0$ or $C = 90^0$

Hence (b) is correct answer.

EXAMPLE – 29

If $A + B = \frac{\pi}{3}$, where $A, B \in R^+$, then the minimum value of secA + secB is equal to

(a) $\frac{2}{\sqrt{3}}$ (b) $\frac{4}{\sqrt{3}}$ (c) $2\sqrt{3}$ (d) none

Solution:

For y = secx, x ϵ [0, π/2), tangent drawn to it any point lies completely below the graph of y = secx, thus

$\frac{\sec A + \sec B}{2} \geq \sec\left(\frac{A+B}{2}\right) \Rightarrow \sec A + \sec B \geq 2.\sec\left(\frac{\pi}{6}\right) = \frac{4}{\sqrt{3}}$

Hence (b) is correct answer.

EXAMPLE – 30

If $\cos x = \frac{2\cos y - 1}{2 - \cos y}$, where x, y ϵ (0,π) then $\tan\frac{x}{2}.\cot\frac{y}{2}$ is equal to

(a) $\sqrt{2}$ (b) $\sqrt{3}$ (c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{\sqrt{3}}$

Solution:

$\cos x = \frac{2\cos y - 1}{2 - \cos y} \Rightarrow \frac{1 - \tan^2\frac{x}{2}}{1 + \tan^2\frac{x}{2}} = \frac{\frac{2(1 - \tan^2 y/2)}{1 + \tan^2 y/2} - 1}{2 - \frac{1 - \tan^2 y/2}{1 + \tan^2 y/2}}$

$\Rightarrow 6\tan^2\frac{y}{2} = 2\tan^2\frac{x}{2} \Rightarrow \tan\frac{x}{2}.\cot\frac{y}{2} = \sqrt{3}$

Hence (b) is correct answer.

EXAMPLE – 31

If $\frac{1 + \sin 2x}{1 - \sin 2x} = \cot^2(a + x)\ \forall\ x \in R\sim\left(n\pi + \frac{\pi}{4}\right)$, n ϵ N. Then 'a' equals to

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}$ (c) $\frac{3\pi}{2}$ (d) none

Solution:

$\frac{1 + \sin 2x}{1 - \sin 2x} = \frac{(\sin x + \cos x)^2}{(\sin x - \cos)^2} = \left(\frac{1 + \cot x}{1 - \cot x}\right)^2$

$\Rightarrow \left(\frac{1 + \tan x}{1 - \tan x}\right)^2 = \left(\tan\left(\frac{\pi}{4} + x\right)\right)^2 = \tan^2\left(\frac{\pi}{4} + x\right)$

$= \cot^2\left(\frac{\pi}{2} + \frac{\pi}{4} + x\right) = \cot^2\left(\frac{3\pi}{4} + x\right)$

Hence (c) is correct answer.

EXAMPLE – 32

If sin α sin β – cos α cos β+1 = 0, then the value of cot α tan β is

(a) –1 (b) 0 (c) 1 (d) none

Solution:

Given sin α sin β – cos α cos β+ 1=0

$\Rightarrow \cos(\alpha+\beta) = -1 \Rightarrow \alpha+\beta = (2n+1)\pi$

$1 + \cot\alpha\tan\beta = 1 + \frac{\cos\alpha}{\sin\alpha}.\frac{\sin\beta}{\cos\beta} = \frac{\sin\alpha\sin\beta + \cos\alpha\sin\beta}{\sin\alpha\cos\beta}$

$= \frac{\sin(\alpha+\beta)}{\sin\alpha\cos\beta} = \frac{\sin((2n+1)\pi)}{\sin\alpha\cos\beta} = 0$

$\Rightarrow$ cot α tan β = –1

Hence (a) is correct answer.

EXAMPLE – 33

The value of $2^{\sin x} + 2^{\cos x}$ is

(a) $\le 2^{1-\frac{1}{\sqrt{2}}}$ (b) $\ge 2^{1-\frac{1}{\sqrt{2}}}$ (c) $\ge 2^{1+\frac{1}{\sqrt{2}}}$ (d) $\le 2^{1+\frac{1}{\sqrt{2}}}$

Solution:

Since AM of two positive quantities ≥ their G.M

$\frac{2^{\sin x}+2^{\cos x}}{2} \ge \sqrt{2^{\sin x}\,.\,2^{\cos x}}$

$= \sqrt{2^{\sin x+\cos x}} = \sqrt{2^{\sqrt{2}\cos\left(x-\frac{\pi}{4}\right)}} \Rightarrow 2^{\sin x} + 2^{\cos x} \ge 2\,.\,2^{1-\frac{1}{\sqrt{2}}}$

Hence (c) is correct answer.

EXAMPLE – 34

The value of

$\left(1+\cos\frac{\pi}{8}\right)\left(1+\cos\frac{3\pi}{8}\right)\left(1+\cos\frac{5\pi}{8}\right)\left(1+cos\frac{7\pi}{8}\right)$ is

equal to:

(a) $\frac{1}{4}$ (b) $\frac{1}{6}$ (c) $\frac{1}{8}$ (d) $\frac{1}{2}$

Solution:

Using $\cos\frac{5\pi}{8} = \cos\left(\frac{\pi}{2}+\frac{\pi}{8}\right) = -\sin\frac{\pi}{8}$

$\Rightarrow \cos\frac{3\pi}{8} = \cos\left(\frac{\pi}{2}-\frac{\pi}{8}\right) = \sin\frac{\pi}{8}$

$\Rightarrow \cos\frac{7\pi}{8} = \cos\left(\pi-\frac{\pi}{8}\right) = -\cos\frac{\pi}{8}$

$\Rightarrow \left(1+cos\frac{\pi}{8}\right)\left(1-cos\frac{\pi}{8}\right)\left(1+sin\frac{\pi}{8}\right)\left(1-sin\frac{\pi}{8}\right)$

$\Rightarrow \left(1-cos^2\frac{\pi}{8}\right)\left(1-sin^2\frac{\pi}{8}\right) = sin^2\frac{\pi}{8}\,cos^2\frac{\pi}{8}$

$\Rightarrow \frac{1}{4}\left(sin\frac{\pi}{4}\right)^2 \Rightarrow \frac{1}{4}.\frac{1}{2} = \frac{1}{8}$

Hence (c) is correct answer.

EXAMPLE – 35

Maximum value of the expression 2 sin x + 4 cos x +3 is

(a) $2\sqrt{5}+3$ (b) $2\sqrt{5}-3$ (c) $\sqrt{5}+3$ (d) none

Solution:

Maximum value of 2 sin x + 4 cos x = $2\sqrt{5}$.

Hence the maximum value of 2 sin x + 4 cos x+3 is $2\sqrt{5}+3$

Hence (a) is correct answer.

TRIGONOMETRIC EQUATION

EXAMPLE – 1

The number of solutions of the equation

|cotx| = cotx + 1/sinx (0 ≤ x ≤ 2π) is

(a) 0 (b) 1 (c) 2 (d) 3

Solution:

If cotx > 0, then 1/sinx = 0 which is not possible. Now if, cotx < 0, then:

– cots = cotx + 1/sinx ⇒ (2cosx + 1)/sinx = 0

⇒ cosx = – ½. ⇒ x = 2nπ ± 2π/3, n ∈ I and 0 ≤ x ≤ 2π, also cotx < 0. ⇒ x = 2π/3

Hence (b) is correct answer.

EXAMPLE – 2

The number of solutions of the equation $\sin^3x\cos x + \sin^2x\cos^2x + \sin x\cos^3x = 1$, in the interval [0, 2π] is

(a) 0 (b) 2 (c) 3 (d) infinite

Solution:

Here: $\sin^3x\cos x + \sin^2x\cos^2x + \sin x\cos^3x = 1$

⇒ $\sin x\cos x(\sin^2x + \sin x\cos x + \cos^2x) = 1$

⇒ sin2x/2 (1 +sin2x/2) = 1

⇒ sin2x(2 + sin2x) = 4

⇒ $\sin^2 2x + 2\sin 2x - 4 = 0$

⇒ $\sin 2x = \frac{-2\pm\sqrt{4+16}}{2} \Rightarrow \sin 2x = -1 \pm \sqrt{5}$.

That is not possible (since – 1 ≤ sin2x ≤ 1)

Hence no solutions.

Hence (a) is correct answer.

EXAMPLE – 3

The value of

$\sin^{-1}\{\cot(\sin^{-1}\sqrt{\frac{(2-\sqrt{3})}{4}}) + \cos^{-1}\sqrt{(12)}/4 + \sec^{-1}(\sqrt{2})\}$ is

(a) 0 (b) π/4 (c) π/6 (d) π/2

Solution:

We have: $\sin^{-1}\{\cot(\sin^{-1}\sqrt{\frac{(2-\sqrt{3})}{4}}) + \cos^{-1}\sqrt{(12)}/4 + \sec^{-1}(\sqrt{2})\}$

$= \sin^{-1}\{\cot(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}) + \cos^{-1}\sqrt{(3)}/2 + \cos^{-1}(1/\sqrt{2})\}$

$= \sin^{-1}\{\cot(\sin^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}}) + \cos^{-1}\frac{\sqrt{3}-1}{2\sqrt{2}})$

$= \sin^{-1}(\cot\pi/2) = \sin^{-1}0 = 0$.

Hence (a) is correct answer.

EXAMPLE – 4

The set of values of x for which $\frac{tan3x-tan2x}{1+tan3x.tan2x} = 1$ is

(a) ϕ (b) {π/4}

(c) {nπ + π/4, n = 1, 2, 3…}

(d) {2nπ + π/4, n = 1, 2, 3,…..}

Solution:

$\frac{tan3x-tan2x}{1+tan3x.tan2x} = 1 \Rightarrow \tan(3x-2x) = 1$

⇒ tanx = 1 ⇒ tanx = tanπ/4 ⇒ x = nπ + π/4.

But for this value of x: tan2x = tan(2nπ + π/2) = ∞.

Hence (a) is correct answer.

EXAMPLE – 5

The value of a fro which the equation $4\text{cosec}^2(\pi(a+x)) + a^2 - 4a = 0$ has a real solution, is

(a) a = 1 (b) a = 2 (c) a = 10 (d) none

Solution:

Here $4[1 + \cot^2\pi(a+x)] + a^2 - 4a = 0$

⇒ $4\cot^2\pi(a+x) + (a-2)^2 = 0$

⇒ $a - 2 = 0$ and $\cot^2\pi(a+x) = 0 \Rightarrow a = 2$.

Hence (b) is correct answer.

EXAMPLE – 6

If $\left(\frac{1+sin\theta-cos\theta}{1+sin\theta+cos\theta}\right)^2 = \lambda\frac{1-cos\theta}{1+cos\theta}$. then λ equals to

(a) –1 (b) 1 (c) 2 (d) – 2

Solution:

We have $\left(\frac{1+sin\theta-cos\theta}{1+sin\theta+cos\theta}\right)^2 = \left(\frac{2sin^2\theta/2+2sin\theta/2cos\theta/}{1+sin\theta+cos\theta}\right)^2$

$= \tan^2\theta/2 = \frac{1-cos\theta}{1+cos\theta}$.

Hence (b) is correct answer.

EXAMPLE – 7

The number of real solutions of

$\tan^{-1}(\sqrt{x(x+1)}) + \sin^{-1}(\sqrt{x^2+x+1}) = \pi/2$ is

(a) 0 (b) 1 (c) 2 (d) infinite

Solution:

The given condition holds if

$\tan^{-1}(\sqrt{x(x+1)}) + \sin^{-1}(\sqrt{x^2+x+1}) = \pi/2$

$\Rightarrow \tan^{-1}(\sqrt{x(x+1)}) - \cos^{-1}(\sqrt{x^2+x+1})$

$\Rightarrow \cos^{-1}(1/\sqrt{x^2+x+1}) = \cos^{-1}(\sqrt{x^2+x+1})$

$\Rightarrow x^2 + x + 1 = 1 \Rightarrow x = 0, -1$.

Hence (c) is correct answer.

EXAMPLE – 8

One of the general value of θ satisfying the equation $\tan^2\theta + \sec 2\theta = 1$ is

(a) 2nπ (b) nπ+ π/4 (c) nπ ± π/3 (d) none

Solution:

The given equation can be written as

$\tan^2\theta + (1 + \tan^2\theta)/(1 - \tan^2\theta) = 1$

$\Rightarrow \tan^2\theta(\tan^2\theta - 3) = 0 \Rightarrow \tan\theta = \pm\sqrt{3}$

Hence (c) is correct answer.

EXAMPLE – 9

The number of roost of the equation x + 2tanx = π/2 in the interval [0, 2π] is,

(a) 1 (b) 3 (c) 2 (d) infinite

Solution:

We have x + 2tanx = π/2 Or tanx = π/4 – x/2

Now the graph of the curve y = tanx and

y = π/4 – x/2, in the interval [0, 2π] intersect at three points.

The abscissa of these three points are the roots of the equation.

Hence (b) is correct answer.

EXAMPLE – 10

sinx + cosx = y^2 – y + a has no value of x for any y if 'a' belongs to

(a) (0, √3) (b) (–√3 , 0)

(c) (–∞, – √3) (d) (√3, ∞)

Solution:

$y^2 - y + a = (y - 1/20)^2 + a - 1/4$

Since: $-\sqrt{2} \le \sin x + \cos x \le \sqrt{2}$, given equation will have no real value of x for any y if a – ¼ > √2. i.e a ∈(√2 + ¼, ∞) ∞a ∈ (√3, ∞) (as √2 + ¼ < √3)

Hence (d) is correct answer.

EXAMPLE – 11

If $\sin^{-1}(x - x^2/2 + x^3/4 - ...) + \cos^{-1}(x^2 - x^4/4 + x^6/6 + ...) = \pi/2$. For 0 < |a| < √2, then x equals

(a) ½ (b) 1 (c) –1/2 (d) – 1

Solution:

The given equation: (using: GP)

$\sin^{-1}(x/(1 + x/2)) + \cos^{-1}(x^2/(1 + x^2/2)) = \pi/2$

$\Rightarrow 2x/(x + 2) = 2x^2/(2 + x^2) \Rightarrow x = 1$.

Hence (b) is correct answer.

EXAMPLE – 12

If the equation tanθ + tan2θ + tanθ.tan2θ = 1, then θ is equal to

(a) $\frac{n\pi}{3} - \frac{\pi}{6}$ (b) $\frac{n\pi}{3} + \frac{\pi}{12}$ (c) $\frac{n\pi}{3} + \frac{\pi}{2}$ (d) none

Solution:

We re-write the equation as

tanθ + tan2θ = 1– tanθ tan2θ

$\Rightarrow \frac{tan\theta\ tan2\theta}{1-tan\theta\ tan2\theta} = 1 \Rightarrow \tan 3\theta = 1 = \tan\frac{\pi}{4}$

$\Rightarrow 3\theta = n\pi + \frac{\pi}{4} \Rightarrow \theta = \frac{n\pi}{3} + \frac{\pi}{12}$

Hence (b) is a correct answer

EXAMPLE – 13

The equation k cos x –3 sin x = k +1 is solvable only if k belongs to th interval

(a) [k, +∞] (b) [–4,4] (c) (–∞,4] (d) none

Solution:

The condition is $\left|\frac{k+1}{\sqrt{k^2+9}}\right| \le 1$, i.e. $\Rightarrow (k + 1)^2 \le k^2 + 9 \Rightarrow k \le 4$

Hence (c) is correct answer.

EXAMPLE – 14

The number of solutions of $\cos\theta + \sqrt{3}\sin\theta = 5$, 0 ≤ θ ≤ 5π, is

(a) 4 (b) 0 (c) 5 (d) none

Solution: $\left|\frac{5}{\sqrt{1^2+(\sqrt{3})^2}}\right| \le 1$ is not true.

So, there is no solution.

Hence (b) is correct answer.

EXAMPLE – 15

If the equation $\sin\theta = -\frac{1}{2}$ and $\tan\theta = \frac{1}{\sqrt{3}}$, the most common general values of θ is

(a) $2n\pi \pm \frac{7\pi}{6}$ (b) $2n\pi - \frac{7\pi}{6}$ (c) $2n\pi + \frac{7\pi}{6}$ (d) none

Solution:

First find the values of θ lying between 0 and 2π and satisfying the two given equation separately. Select the value of θ which satisfies both the equations, then generalize it.

$\text{Sin}\theta = -\frac{1}{2} \Rightarrow \theta = \frac{7\pi}{6}$ or $\frac{11\pi}{6}$

$\tan\theta = \frac{1}{\sqrt{3}} \Rightarrow \theta = \frac{\pi}{6}$ or $\frac{7\pi}{6}$ [values between 0 and 2π]

Common value of $\theta = \frac{7\pi}{6}$

The required solution is $\theta = 2n\pi + \frac{7\pi}{6}$.

Hence (c) is correct answer.

EXAMPLE – 16

If $x \in \left[-\frac{5\pi}{2}, \frac{5\pi}{2}\right]$, the greatest positive solutions of $1 + \sin^4 x = \cos^2 3x$ is

(a) π (b) 2π (c) $\frac{5\pi}{2}$ (d) none

Solution:

$\sin^2 3x + \sin^4 x = 0$

Or $\sin^2 x\{(3 - 4\sin^2 x)^2 + \sin^2 x\} = 0$

So, $\sin x = 0 \Rightarrow x = n\pi$

Hence (b) is correct answer.

EXAMPLE – 17

The number of solutions of the equation $x^3 + x^2 + 4x + 2\sin x = 0$ in $0 \le x \le 2\pi$, is

(a) zero (b) one (c) two (d) four

Solution:

Here, $x^3 + (x + 2)^2 + 2\sin x = 4$

Clearly, x = 0 satisfies the equation

If $0 < x \le \pi$, $x^3 + (x + 2)^2 + 2\sin x > 4$.

If $\pi < x \le 2\pi$,

$x^3 + (x + 2)^2 + 2\sin x > 27 + 25 - 2$

So, x = 0 is the only solution.

Hence (b) is correct answer.

EXAMPLE – 18

The number of real solutions of $\sin e^x . \cos e^x = 2^{x-2} + 2^{-x-2}$ is

(a) zero (b) one (c) two (d) infinite

Solution:

Here, $\frac{1}{2}\sin(2e^x) = \frac{1}{4}(2^x + 2^{-x}) \ge \frac{1}{4} . 2 = \frac{1}{2}$; so, $\sin(2e^x) \ge 1$

But $\sin(2e^x) > 1$. Therefore, $\sin(2e^x) = 1$

But equality can hold when $2^x = 2^{-x} = 1$, i.e, x = 0

Then $\sin(2.e^0) = 1$, which is not true.

Hence (a) is correct answer.

EXAMPLE – 19

If equation $\sin^4 x = 1 + \tan^8 x$, then x is

(a) ϕ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Solution:

$\sin^4 x = 1 + \tan^8 x$ only when

$\text{Sin}^4 x = 1$ and $1 + \tan^8 x = 1 \Rightarrow \sin^2 x = 1$ and $\tan^8 x = 0$

Which is never possible, since sin x and tan x vanish simultaneously. Therefore, the given equation has no solution.

Hence (a) is correct answer.

EXAMPLE – 20

If $\sin^2 x + \cos^2 y = 2\sec^2 z$, then

(a) $x = (2m+1)\frac{\pi}{2}$, $y = n\pi$ and $z = t\pi$

(b) $x = (m+1)\frac{\pi}{2}$, $y = 2n\pi$ and $z = t\pi$

(c) $x = (2m-1)\frac{\pi}{2}$, $y = n\pi$ and $z = 2t\pi$ (d) none of the above

Solution:

$\sin^2 x + \cos^2 y = 2\sec^2 z$ only when

$\sin^2 x = 1$, $\cos^2 y = 1$, $\sec^2 z = 1$

$\Rightarrow \cos^2 x = 0$, $\sin^2 y = 0$, $\cos^2 z = 1$

$\Rightarrow \cos x = 0$, $\sin y = 0$, $\sin z = 0$

$x = (2m + 1)\frac{\pi}{2}$, $y = n\pi$ and $z = n\pi$, where m, n, t are intehers.

Hence (a) is correct answer.

EXAMPLE – 21

The equation $p\cos x - q\sin x = r$ admits of a solution for x only if

(a) r < max{p,}

(b) $-\sqrt{p^2 + q^2} < r < \sqrt{p^2 + q^2}$

(c) $r^2 = p^2 + q^2$ (d) none

Solution:

Let, $p = a\cos\theta$ and $-q = a\sin\theta$

So, $a(\cos\theta\cos x + \sin\theta\sin x) = r \Rightarrow a\cos(x - \theta) = r$

Thus the condition for solution is, $-a \le r \le a$. Where,

$a = \sqrt{p^2 + q^2}$

⇒ Thus equation $p\cos x - q\sin x = r$ admits solution for $-\sqrt{p^2 + q^2} \le r \le +\sqrt{p^2 + q^2}$

Hence (b) is correct answer.

EXAMPLE – 22

The number of values of θ in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, satisfying the equation $(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$ is

(a) 2 (b) 3 (c) 4 (d) none

Solution:

$(1 - \tan\theta)(1 + \tan\theta)\sec^2\theta + 2^{\tan^2\theta} = 0$

$\Rightarrow (1 - \tan^2\theta)(1 + \tan^2\theta) + 2^{\tan^2\theta} = 0$

$\Rightarrow 1 + 2^{\tan^2\theta} = \tan^4\theta$. By observation, we have $\tan^2\theta = 3$

$\Rightarrow \theta = n\pi \pm \left(\frac{\pi}{3}\right)$

Moreover there will be values of θ, satisfying, $3 < \tan^2\theta < 4$ and satisfying the given equation as if $f(x) = x^2 - 2^x - 1$, then $f(3^+) f(4^-) < 0$

So the number of values of θ is 4.

Hence (c) is correct answer.

EXAMPLE – 23

If $\sin^2 x - 2\sin x - 1 = 0$ has exactly four different solutions in $x \in [0, n\pi]$, then minimum value of n can be, ($n \in N$)

(a) 2 (b) 3 (c) 4 (d) none

Solution:

$\sin^2 x - 2\sin x - 1 = 0 \Rightarrow (\sin x - 1)^2 = 2 \Rightarrow \sin x - 1 = \pm\sqrt{2}$

$\Rightarrow \sin x = 1 - \sqrt{2}$ as $\sin x \not> 1$.

There are 2 solutions in $[0, 2\pi]$ and two more in $[2\pi, 4\pi]$.

Thus $n_{min} = 4$

Hence (c) is correct answer.

EXAMPLE – 24

If $\begin{vmatrix} 1+\sin^2 & \cos^2 & 4\sin 4\theta \\ \sin^2 & 1+\cos^2 & 4\sin 4\theta \\ \sin^2 & \cos^2 & 1+4\sin 4\theta \end{vmatrix} = 0$,

such that $0 \le \theta \frac{\pi}{2}$, then θ is

(a) $\frac{7\pi}{12}$ (b) $\frac{7\pi}{24}, \frac{11\pi}{24}$ (c) $\frac{7\pi}{12}, \frac{11\pi}{12}$ (d) none

Solution:

Operating $R_1 \to R_1 - R_3$, $R_2 \to R_2 - R_3$ and then $C_3 \to C_2 + C_1$, we get

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ \sin^2\theta & \cos^2\theta & 1+\sin^2\theta+4\sin 4\theta \end{vmatrix} = 0$$

On expanding, we get

$1 + \sin^2\theta + 4\sin 4\theta + \cos^2\theta = 0$

$\Rightarrow 2 + 4\sin 4\theta = 0 \Rightarrow \sin 4\theta = -\frac{1}{2} = \sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow 4\theta = n\pi + (-1)^n\left(-\frac{\pi}{6}\right) \Rightarrow \theta = \frac{n\pi}{4} - (-1)^n\frac{\pi}{24}$

For $0 \le \theta \frac{\pi}{2}$, we have: $\theta = \frac{7\pi}{24}, \frac{11\pi}{24}$

Hence (b) is correct answer.

EXAMPLE – 25

The value of θ for which the following system has a non-trival solution :

$(\sin 3\theta)x - y + z = 0$

$(\cos 2\theta)x + 4y + 3z = 0$

$2x + 7y + 7z = 0$, are

(a) $n\pi, n\pi + (-1)^n\frac{\pi}{6}$ (b) $n\pi, n\pi - (-1)^n\frac{\pi}{6}$

(c) $2n\pi, n\pi + (-1)^n\frac{\pi}{6}$ (d) $n\pi, n\pi + (-1)^n\frac{\pi}{3}$

Solution:

The system has non-trival solutions, if $\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$.

On expanding we get

$7\sin 3\theta + 14\cos 2\theta - 14 = 0$

$\Rightarrow \sin 3\theta + 2\cos 2\theta - 2 = 0 \Rightarrow \sin\theta \,.\, [4\sin^2\theta + 4\sin\theta - 3] = 0$

$\Rightarrow \sin\theta = 0 \Rightarrow \theta = n\pi$ Or $4\sin^2\theta + 4\sin\theta - 3 = 0$

$\Rightarrow (2\sin\theta - 1)(2\sin\theta + 3) = 0$

$\Rightarrow 2\sin\theta - 1 = 0 \Rightarrow \sin\theta = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right)$

Or $4\sin^2\theta + 4\sin\theta - 3 = 0$, which is not possible

$\Rightarrow \theta = n\pi + (-1)^n\frac{\pi}{6} \Rightarrow \theta = n\pi$ or $n\pi + (-1)^n\frac{\pi}{6}$

Hence (a) is correct answer.

EXAMPLE – 26

A triangle ABC is such that $\sin(2A + B) = \frac{1}{2}$

If, A , B, Care in A.P., then the values of A, B, C are

(a) $\frac{\pi}{3}, \frac{\pi}{4}, \frac{5\pi}{12}$ (b) $\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{6}$

(c) $\frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{4}$ (d) none

Solution:

$\sin(2A + B) = \frac{1}{2} = \sin\left(\frac{\pi}{6}\right) \Rightarrow 2A + B = n\pi + (-1)^n\frac{\pi}{6}$

Also, $A + B + C = \pi$ and $2B = A + C$

$\Rightarrow 3B = \pi \Rightarrow B = \frac{\pi}{3}$

From (1), for n =1, $2A + B = \frac{5\pi}{6} \Rightarrow A = \frac{\pi}{4} \Rightarrow C = \frac{5\pi}{12}$

Hence (a) is correct answer.

EXAMPLE – 27

Total number of integral values of 'n' so that $\sin x(\sin x + \cos x) = n$ has at least one solution , is

(a) 2 (b) 1 (c) 3 (d) zero

Solution:

$\sin x(\sin x + \cos x) = n \Rightarrow \sin^2 x + \sin x \,.\, \cos x = n$

$\Rightarrow \frac{1-\cos 2x}{2} + \frac{\sin 2x}{2} = n \Rightarrow \sin 2x - \cos 2x = 2n-1$

$\Rightarrow -\sqrt{2} \le 2n-1 \le \sqrt{2} \Rightarrow \frac{1-\sqrt{2}}{2} \le n \le \frac{1+\sqrt{2}}{2} \Rightarrow n = 0,1.$

Hence (a) is correct answer.

EXAMPLE – 28

The equation $\tan^4 x - 2\sec^2 x + a^2 = 0$ will have at least one solution, if

(a) $|a| \le 4$ (b) $|a| \le 2$ (c) $|a| \le 3$ (d) none

Solution:

$\tan^4 x - 2\sec^2 x + a^2 = 0 \Rightarrow \tan^2 x - 2(1 + \tan^2 x) + a^2 = 0$

$\Rightarrow \tan^4 x - 2\tan^2 x + 1 = 3 - a^2$

$\Rightarrow (\tan^2 x - 1)^2 = 3 - a^2 \Rightarrow 3 - a^2 \ge 0 \Rightarrow a^2 \le 3 \Rightarrow |a| \le \sqrt{3}$

Hence (c) is correct answer.

PROPERTIES AND SOLUTION OF TRIANGLES

EXAMPLE – 1

If K be the perimeter of the ΔABC, then $b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2}$ is equal to

(a) k (b) 2k (c) $\frac{k}{2}$ (d) none

Solution:

$b\cos^2\frac{C}{2} + c\cos^2\frac{B}{2} = \frac{b}{2}(1 + \cos C) + \frac{C}{2}(1 + \cos B)$

$= \frac{b}{2} + \frac{C}{2} + \frac{1}{2}(b\cos C + c\cos B) = \frac{a+b+c}{2} = \frac{k}{2}$

Hence (c) is correct answer.

EXAMPLE – 2

In a ΔABC, $\cot\frac{A-B}{2}\ .\ \tan\frac{A+B}{2}$ is equal to

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{a(a-b)}{b(a+b)}$ (d) none

Solution:

$$\frac{\tan\left(\frac{A+B}{2}\right)}{tan\left(\frac{A-B}{2}\right)} = \frac{(a+b).tan\left(\frac{A+B}{2}\right)}{(a-b).\cot\frac{C}{2}} = \frac{a+b}{a-b}$$

Hence (a) is correct answer.

EXAMPLE – 3

In a ΔABC, A : B : C = 3:5:4.

Then a + b + $c\sqrt{2}$ is equal to:

(a) 2b (b) 2c (c) 3b (d) 3a

Solution:

Clearly, A = 45^0, B = 75^0, C = 60^0.

So, $\frac{a}{sin45^0} = \frac{b}{sin75^0} = \frac{C}{sin60^0}$ = 2R

So, a = $\sqrt{2}$ R, b = $\frac{\sqrt{3}+1}{\sqrt{2}}$ R, C = $\sqrt{3}$R

Now, a + b + $c\sqrt{2}$ = $\left[\sqrt{2}+\frac{\sqrt{3}+1}{\sqrt{2}}+\sqrt{6}\right]$ R = 3b

Hence (c) is correct answer.

EXAMPLE – 4

If in a ΔABC , $a^2\cos^2A = b^2 + c^2$, then

(a) A < $\frac{\pi}{4}$ (b) $\frac{\pi}{4}$ < A < $\frac{\pi}{2}$ (c) A > $\frac{\pi}{2}$ (d) A = $\frac{\pi}{2}$

Solution:

cosA = $\frac{b^2+C^2-a^2}{2bc} = \frac{a^2cos^2A-a^2}{2bc} = \frac{a^2(cos^2A-1)}{2bc}$ < 0. So, A > $\frac{\pi}{2}$.

Hence (c) is correct answer.

EXAMPLE – 5

Two sides of a triangle are $2\sqrt{2}$ cm and $2\sqrt{3}$cm and the angle opposite to the shorter side of the two is $\frac{\pi}{4}$. The largest possible length of the third side is

(a) $\sqrt{2}\left(\sqrt{3}+1\right)$ cm (b) $\left(\sqrt{6}+\sqrt{2}\right)$cm

(c) ($\sqrt{6}$ –$\sqrt{2}$) (d) none

Solution:

$$\cos\frac{\pi}{4} = \frac{(2\sqrt{3})^2+X^2-(2\sqrt{2})^2}{2\ .2\sqrt{3}\ .\ X}$$

Or $\frac{4\sqrt{3}}{\sqrt{2}}$ x = x^2 + 4 or $x^2 - \frac{4\sqrt{3}}{\sqrt{2}}$ x + 4 = 0

So, x = $\sqrt{6} \pm \sqrt{2}$, neglecting $\sqrt{6}-\sqrt{2}$for greater side.

Hence (a) is correct answer.

EXAMPLE – 6

The area of a ΔABC is $a^2 - (b - c)^2$. Then tan A is equal to

(a) $\frac{4}{3}$ (b) $\frac{3}{4}$ (c) $\frac{8}{15}$ (d) none

Solution:

Here, $\Delta = a^2-(b-c)^2 \Rightarrow \Delta$ = (2s–2c) (2s–2b) = 4 (s–b)(s–c)

So, $\frac{1}{4} = \frac{(s-b)(s-c)}{\Delta} = \tan\frac{A}{2} \Rightarrow \tan A = \frac{2\tan\frac{A}{2}}{1-tan^2\frac{A}{2}} = \frac{2\times\frac{1}{4}}{1-\left(\frac{1}{4}\right)^2} = \frac{8}{15}$

Hence (c) is correct answer.

EXAMPLE – 7

In any triangle ABC, the sides are 6cm, 10cm and 14cm. then the triangle is obtuse-angled with the obtuse angle equal to:

(a) 180^0 (b) 90^0 (c) 120^0 (d) 160^0

Solution:

Let a = 14, b = 10 , c = 6 $\Rightarrow$ s = 15. The largest angle is opposite the longest side. Hence,

$\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}} = \sqrt{\frac{5\times9}{15}} = \sqrt{3} \Rightarrow \frac{A}{2} = 60^0 \Rightarrow A = 120^0$

Alter: cosA = $\frac{a^2+c^2-a^2}{2bc} = \frac{100+36-196}{120} = -\frac{1}{2} \Rightarrow A = 120^0$

Hence (c) is correct answer.

EXAMPLE – 8

Two sides of a triangle are $\sqrt{3}$ –1 and $\sqrt{3}$ + 1 units and their included angle is 60^0. Then the third side of the triangle is:

(a) $\sqrt{6}$ (b) $\sqrt{8}$ (c) 1+$\sqrt{6}$ (d) $\sqrt{5}$

Solution:

$a^2 = b^2 + c^2 - 2bc\cos A$

$= (\sqrt{3}+1)^2 + (\sqrt{3}-1)^2 - 2(\sqrt{3}+1)(\sqrt{3}-1)\cos60^0$

$= 8 - 4\ .\ \frac{1}{2} = 6 \Rightarrow a = \sqrt{6}$

Hence (a) is correct answer.

EXAMPLE – 9

If in a triangle ABC, B = 30^0, C = 60^0 and a = 6 cm, then the rest sides of the triangle are

(a) 3, $3\sqrt{3}$ (b) 3, 3 (c) 2, $2\sqrt{3}$ (d) 2, $3\sqrt{2}$

Solution:

A = 180^0– (B + C) = 90^0

Also, $\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$

$\Rightarrow$ b = asinB = 6 $\times\frac{1}{2}$ = 3, c = asinC = $3\sqrt{3}$

Hence (a) is correct answer.

EXAMPLE – 10

The ratio of the sides of a triangle whose interior angles are 30^0 , 60^0 , 90^0 is

(a) 1:3:2 (b) 1:2:3 (c) 2:1:4 (d) 1:$\sqrt{3}$:2

Solution:

a:b:c = sinA:sinB:sinC = $\frac{1}{2} : \frac{\sqrt{3}}{2}$: 1 = 1: $\sqrt{3}$: 2

Hence (d) is correct answer.

EXAMPLE – 11

If in a ΔABC,

$\sin^3A + \sin^3B + \sin^3C$ =3 sin A. sinB . sinC, then the value of the determinant $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix}$ is

(a) 0 (b) $(a + b + c)^3$

(c) (a + b + c)(ab + bc + ca) (d) none

Solution:

$\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = (a + b + c) \begin{vmatrix} 1 & 1 & 1 \\ b & c & a \\ c & a & b \end{vmatrix}$

$= (a + b + c)(bc + ca + ab - a^2 - b^2 - c^2)$

$= -(a^3 + b^3 + c^3 - 3bc)$

$= -8R^3(\sin^3A + \sin^3B + \sin^3C - 3\sin A\sin B\sin C)$

= 0, from the question.

Hence (a) is correct answer.

EXAMPLE – 12

In a ΔABC, a =1 and the perimeter is six times the A.M. of the sines of the angles. The measure of ∠A is

(a) $\frac{\pi}{3}$ (b) $\frac{\pi}{2}$ (c) $\frac{\pi}{6}$ (d) $\frac{\pi}{4}$

Solution:

$1 + b + c = 6.\frac{\sin A \sin B \sin C}{3} = 2\left(\frac{1}{2R} + \frac{b}{2R} + \frac{c}{2R}\right) = \frac{1}{R}(1+b+c)$

So, R = 1 (So, $1 + b + c \neq 0$)

So, $\frac{1}{\sin A} = 2R = 2 \Rightarrow A = \frac{\pi}{6}$.

Hence (c) is correct answer.

EXAMPLE – 13

If in a triangle, R and r are the circumradius and inradius respectively, then the H.M. of the exradii of the triangle is

(a) 3r (b) 2R (c) R + r (d) none

Solution:

$\sum\frac{1}{r_1} = \sum\frac{s-a}{\Delta} = \frac{3s-(a+b+c)}{\Delta} = \frac{s}{\Delta}$

So, H.M. of exradii $= \frac{1}{\frac{\sum 1/r_1}{3}} = \frac{3}{s/\Delta} = \frac{3\Delta}{s} = 3r$

Hence (a) is correct answer.

EXAMPLE – 14

If the ex-radii of a triangle are in H.P., then the corresponding sides are in

(a) A.P (b) G.P (c) H.P (d) none

Solution:

$\frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}$ are in A.P $\Rightarrow \frac{s-a}{\Delta}, \frac{s-b}{\Delta}, \frac{s-c}{\Delta}$ are in A.P

$\Rightarrow$ s – a , s – b , s – c are in A.P. $\Rightarrow$ – a, – b, – c are in A.P

Hence (a) is correct answer.

EXAMPLE – 15

In a ΔABC, the inradius and three exradii are r, r_1, r_2 *and* r_3 respectively. In usual notations the value of $r.r_1.r_2.r_3$ is equal to

(a) 2Δ (b) Δ^2 (c) $\frac{abc}{4R}$ (d) none

Solution:

$r.r_1.r_2.r_3 = \frac{\Delta}{s}.\frac{\Delta}{s-a}.\frac{\Delta}{s-b}.\frac{\Delta}{s-c} = \frac{\Delta^4}{\Delta^2} = \Delta^2$

Hence (b) is correct answer.

EXAMPLE – 16

In aright angled triangle R is equal to

(a) $\frac{s+r}{2}$ (b) $\frac{s-r}{2}$ (c) s – r (d) $\frac{s-r}{a}$

Solution:

In a right angled triangle, the circumcentre lies on the hypotenuse $\Rightarrow R = \frac{a}{2}$, where $A = 90^0$,

Also, $r = (s - a)\tan\frac{A}{2} = (s - a)\tan 45^0 = (s - a) = s - 2R$

$\Rightarrow r + 2R = s \Rightarrow R = \frac{s-r}{2}$

Hence (b) is correct answer.

EXAMPLE – 17

The exradii r_1, r_2, r_3 ΔABC are in H.P then its side a, b, c are in

(a) G.P (b) H.P (c) A.P (d) none

Solution:

r_1, r_2, r_3 are in H.P $\Rightarrow \frac{2}{r_1} = \frac{1}{r_2} = \frac{1}{r_3}$

$\Rightarrow 2(\frac{s-b}{\Delta}) = \frac{s-a}{\Delta} + \frac{s-c}{\Delta} \Rightarrow 2s - 2b = 2s - a - c$

$\Rightarrow$ 2b = a + c hence, a, b, c are in A.P.

Hence (c) is correct answer.

EXAMPLE – 18

If the data given to construct a triangle ABC is a = 5 b = 7, $\sin A = \frac{3}{4}$, then it is possible to construct

(a) only 1 triangle (b) two triangle

(c) infinitely many triangles (d) no Δ

Solution:

We have: sinA/a = ainB/b $\Rightarrow \frac{\frac{3}{4}}{5}$ = sinB/7

$\Rightarrow \sin B = \frac{21}{20} > 1$, which is not possible.

Thus no triangle is possible

Hence (d) is correct answer.

EXAMPLE – 19

The side of triangle are 3x + 4y, 4x + 3y and 5x + 5y units, where x, y > 0. The triangle is

(a) right angle (b) equilateral triangle

(c) obtuse triangle (d) none

Solution:

Let a = 3x + 4y, b = 4x + 3y and c = 5x + 5y.

Clearly, C is the largest side and then the largest angle C is given by $\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{2x}{2(12x^2+25xy+12y^2)} < 0$

$\Rightarrow$ c is an obtuse angle.

Hence (c) is correct answer.

EXAMPLE – 20

If p is a point on the altitude AD of the triangle ABC such that $\angle$ CDP = $\frac{B}{3}$, then AP is equal to

(a) 2a sin$\frac{C}{3}$ (b) 2bsin$\frac{C}{3}$

(c) 2c sin$\frac{B}{3}$ (d) 2c sin$\frac{C}{3}$

Solution:

$\angle BEA = 90^0 + \frac{B}{3}$, A + B = $\frac{2b}{3}$

In triangle ABP, (by sine rule)

$\Rightarrow \frac{AP}{\sin\frac{2b}{3}} = c/\sin(90+\frac{B}{3}) = \frac{C}{\cos(B/3)}$

$\Rightarrow AP = \frac{c\sin(2B/3)}{\cos(B/3)} = \frac{2c\sin\left(\frac{B}{3}\right)\cos(B/3)}{\cos(B/3)} \Rightarrow AP = 2c\sin\left(\frac{B}{3}\right)$

Hence (c) is correct answer.

EXAMPLE – 21

If in a triangle ABC, $\angle B = 90^0$, then $\tan^2\left(\frac{A}{2}\right)$ is

(a) $\frac{b-c}{b+c}$ (b) $\frac{b+c}{b-c}$ (c) $\frac{b-2c}{b+c}$ (d) none

Solution:

$\cos A = \frac{c}{b} \Rightarrow \frac{1-tan^2\frac{A}{2}}{1+tan^2\frac{A}{2}} = \frac{c}{b} \Rightarrow \frac{\left(1+tan^2\frac{A}{2}\right)-\left(1-tan^2\frac{A}{2}\right)}{\left(1+tan^2\frac{A}{2}\right)+\left(1-tan^2\frac{A}{2}\right)} \frac{(b-c)}{(b+c)}$

(by componendo and dividendo rule)

Hence $tan^2\frac{A}{2} = \frac{b-c}{b+c}$

Hence (a) is correct answer.

EXAMPLE – 22

If in a triangle ABC,

$\frac{2cosA}{a} + \frac{cosB}{b} + \frac{2cosC}{c} = \frac{a}{bc} + \frac{b}{ac}$, then b^2+c^2 is equal to

(a) a^2 (b) ac (c) bc (d) none

Solution:

We have, $\frac{2\cos A}{a} + \frac{cosB}{b} + \frac{2cosC}{c} = \frac{a}{bc} + \frac{b}{ac}$

Multiplying both sides by abc

$\Rightarrow$ 2bccosA + accosB + 2abcosC = $a^2 + b^2$

$(b^2 + c^2 - a^2) + \frac{(c^2+a^2-b^2)}{2} + (a^2 + b^2 - c^2)$

$= a^2 + b^2$

$\Rightarrow (c^2 + a^2 - b^2) = 2a^2 - 2b^2 \Rightarrow b^2 + c^2 = a^2$

Hence (a) is correct answer.

EXAMPLE – 23

If A,B and C are angles of triangle such that angle A is obtuse , then tanB.tanC will be less than that angle A is obtuse , then tanB.tanC will be less than

(a) $\frac{1}{\sqrt{3}}$ (b) $\frac{\sqrt{3}}{2}$ (c) 1 (d) none

Solution:

Since A is obtuse angle. Then $90^0 < A < 180^0$

$\Rightarrow 90^0 > B + C > 0 \Rightarrow B + C < 90^0 \Rightarrow B < 90^0 - C$

So, tanB < tan $(90^0 - C)$

tanB < cotC $\Rightarrow$ tanB tanC < 1

Hence (c) is correct answer.

EXAMPLE – 24

If the sides of the triangle ABC are in A.P and a is the smallest side, then cos A equals to

(a) $\frac{3c-4b}{2c}$ (b) $\frac{3c-4b}{2b}$ (c) $\frac{4c-3b}{2c}$ (d) none

Solution:

Since sides of the triangle are in A.P i.e. a, b, c are in A.P and let a < b < c, $\Rightarrow$ 2b = a + c

Now: $\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{b^2+c^2-(2b-c)^2}{abc}$

$= \frac{b^2+c^2-4b^2-c^2+4bc}{2bc} = \frac{4bc - 3b^2}{2bc}$

Hence (a) is correct answer.

EXAMPLE – 25

If a, b, c and d are the sides of a quadrilateral, then the minimum value of $\frac{a^2+b^2+c^2}{d^2}$ is

(a) 1 (b) $\frac{1}{2}$ (c) $\frac{1}{3}$ (d) $\frac{1}{4}$

Solution:

Since $(a - b)^2 + (b - c)^2 + (c - a)^2 \geq 0$

$\Rightarrow 2(a^2 + b^2 + c^2) \geq 2ab + 2bc + 2ca$

$\Rightarrow 3(a^2 + b^2 + c^2) \geq a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$\Rightarrow 3(a^2 + b^2 + c^2) \geq (a + b + c)^2 > d^2$

$\Rightarrow 3(a^2 + b^2 + c^2) > d^2 \Rightarrow \frac{a^2+b^2+c^2}{d^2} > \frac{1}{3}$

Hence the minimum value of $\frac{a^2+b^2+c^2}{d^2} > \frac{1}{3}$.

Hence (c) is correct answer.

EXAMPLE – 26

If A = 30^0, a = 7, b = 8 in ΔABC, then B has

(a) one solution (b) two solution

(c) no solution (d) none

Solution:

We have: a/sinA = b/sinB

$\Rightarrow$ sinB = bsinA/a = 8sin30/7 = 4/7.

So, that b > a > bsinA.

Hence angle B has two values given by sinB = 4/7.

Hence (b) is correct answer.

EXAMPLE – 27

In ΔABC, a = 5, b = 4 and cos(A– B) = 31/32. Then side c is

(a)6 (b) 7 (c) 9 (d) none

Solution:

We have: $\tan(A - B)/2 = \sqrt{\frac{1-\cos(A-B)}{1+\cos(A+B)}} = \sqrt{\frac{1-\frac{31}{32}}{1+\frac{31}{32}}} = 1/\sqrt{63}$.

$\Rightarrow \frac{a-b}{a+b} cot\frac{C}{2} = \frac{1}{\sqrt{63}} \Rightarrow \frac{1}{9} cot\frac{C}{2} = \frac{1}{\sqrt{63}}$

$\Rightarrow \tan\frac{C}{2} = \frac{\sqrt{7}}{3}$, $\cos C = \frac{1-\tan^2\frac{C}{2}}{1+\tan^2\frac{C}{2}} = \frac{1-7/9}{1+7/9} = \frac{1}{8}$.

Since: $c^2 = a^2 + b^2 - 2bc\cos C$,

$c^2 = 25 + 16 - 40\times\frac{1}{8} = 36 \Rightarrow c = 6$.

Hence (a) is correct answer.

EXAMPLE – 28

In a triangle the angles are in AP and the lengths of the two larger sides are 10 and 9 respectively. Then the length of the third side can be

(a) $5 \pm \sqrt{6}$ (b) 0.7 (c) $3 - \sqrt{6}$ (d) none

Solution:

Let the angles be $A = x - d$, $B = x$ and $C = x + d$. Then $B = 60^0$. (By AP)

$\Rightarrow$ two larger angles are B and C. hence $b = 9$, $c = 10$. Now $\cos B = (c^2 + a^2 - b^2)/2ac$

$\Rightarrow ½ = \frac{100+a^2-81}{20a} \Rightarrow a^2 - 10a + 19 = 0 \Rightarrow a = 5 \pm \sqrt{6}$.

Hence (a) is correct answer.

EXAMPLE – 29

If in ΔABC, $\angle A = 90^0$ and c, sinB, cosB are rational numbers, then

(a) a is rational and b is irrational

(b) a is irrational and b is rational

(c) a and b both are rational (d) none

Solution:

Here $C = 90^0 - B$ and $\sin C = \cos B$.

Also, $a = b/\sin B = c/\sin C = c/\cos B \Rightarrow a, b$ are rational.

Hence (c) is correct answer.

EXAMPLE – 30

If P is point on the altitude AD of the ΔABC such that $\angle CBP = B/3$, then AP is equal to

(a) $2a\sin(C/2)$ (b) $2b\sin(C/3)$

(c) $2c\sin(B/3)$ (d) $2c\sin(C/3)$

Solution:

Here $\angle BPA = 90^0 + B/3$, $\angle ABP = 2B/3$.

In ΔABP: $AP/\sin(2B/3) = c/\sin(90 + B/3) = c/\cos(B/3)$.

$\Rightarrow AP = c\sin(2B/3)/\cos(B/3) = 2c\sin(B/3)\cos(B/3)/\cos(B/3)$

$\Rightarrow AP = 2c\sin(B/3)$.

Hence (c) is correct answer.

EXAMPLE – 31

In any triangle ABC, the least value of $(\sin^2A + \sin A + 1)/\sin A$ is

(a) 3 (b) $3\sqrt{3}$ (c) 9 (d) none

Solution:

We have $(\sin^2A + \sin A + 1)/\sin A = \sin A + 1/\sin A + 1$

$= (\sqrt{(\sin A)} - 1/\sqrt{(\sin A)})^2 + 3) \geq 3$.

Hence (a) is correct answer.

EXAMPLE – 32

If A is the area and 2s the sum of the sides of a triangle PQR, then

(a) $A \leq s^2/5\sqrt{3}$ (b) $A \leq s^2/3\sqrt{3}$ (c) $A \leq s^2/\sqrt{3}$ (d) none

Solution:

We have $2s = a + b + c$,

$A^2 = s(s - a)(s - b)(s - c)$. Now:

$AM \geq GM \Rightarrow \frac{s+(s-a)+(s-b)+(s-c)}{4} \geq [s(s - a)(s - b)(s - c)]^{3/4}$.

$\Rightarrow \frac{4s-2s}{4} \geq [A^2]^{3/4} \Rightarrow s/2 \geq A^{1/2} \Rightarrow A \leq s^2/4$.

Also $\frac{(s-a)(s-b)(s-c)}{3} \geq [(s - a)(s - b)(s - c)]^{1/3}$

$\Rightarrow s/3 \geq [A^2/s]^{1/3}$ or $A^2/s \leq s^3/27 \Rightarrow A \leq s^2/3\sqrt{3}$.

Hence (b) is correct answer.

EXAMPLE – 33

In a ΔABC, if $r_1 > r_2 > r_3$, then

(a) $a > b > c$ (b) $a < b < c$

(c) $a > b$ and $b < c$ (d) $a < b$ and $b > c$.

Solution:

We have: $r_1 > r_2 > r_3 \Rightarrow \Delta/(s-a) > \Delta/(s-b) > \Delta/(s-c)$

$\Rightarrow (s - a)/\Delta < (s - b)/\Delta < (s - c)/\Delta$

$\Rightarrow s - a < s - b < s - c \Rightarrow a > b > c$

Hence (a) is correct answer.

EXAMPLE – 34

If A, B and C are angles of a triangle ABC such that $\angle A$ is obtuse, then tanB.tanC will be less than

(a) $1/\sqrt{3}$ (b) $\sqrt{3}/2$ (c) 1 (d) none

Solution:

Since A is obtuse angles.

$90 < A < 180 \Rightarrow 90 < 180 - (B + C) < 180$

$\Rightarrow -90 > B + C - 180 > -180 \Rightarrow 90 > B + C > 0$

$\Rightarrow B + C < 90 \Rightarrow B < 90 - C$

$\Rightarrow \tan B < \tan(90 - C) \Rightarrow \tan B < \cot C \Rightarrow \tan B.\tan C < 1$.

Hence (c) is correct answer.

EXAMPLE – 35

If the sides of a triangle ABC are in AP and is the smallest side, then cosA equals

(a) $(3c - 4b)/2c$ (b) $(3c - 4b)/2b$

(c) $(4c - 3b)/2c$ (d) none

Solution:

Since the sides of the triangle are in AP

i.e. a, b, c are in AP and let $a < b < c$, $2b = a + c$. Now,

$\cos A = (b^2 + c^2 - a^2)/2bc = \frac{b^2+c^2-(2b-c)^2}{2bc}$

$= \frac{b^2+c^2-4b^2-c^2+4bc}{2bc} = \frac{4bc-3b^2}{2bc} = \frac{4c-3b}{2c}$.

Hence (c) is correct answer.

EXAMPLE – 36

Given $b = 2$, $c = \sqrt{3}$, $\angle A = 30^0$, then the in-radius of ΔABC is

(a) $\frac{\sqrt{3}-1}{2}$ (b) $\frac{\sqrt{3}+1}{2}$ (c) $\frac{\sqrt{3}-1}{4}$ (d) none

Solution:

We have b = 2, c = √3 and ∠A = 30^0,

Now: a = $\sqrt{b^2 + c^2 - bccosA}$

$= \sqrt{4 + 3 - 2.2\sqrt{3}.\frac{\sqrt{3}}{2}} = 1.$

⇒ s = (3 +√3)/2. Also, Δ = $\frac{1}{2}bcsinA = \frac{\sqrt{3}}{2}$.

Now r = Δ/s = 1/(√3 + 1) = (√3 – 1)/2.

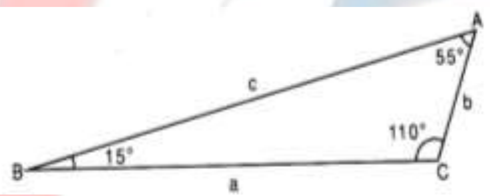
Hence (a) is correct answer.

EXAMPLE – 37

In a Δ, ∠A = 55^o, ∠B = 15^o, ∠C = 110^o. then $c^2 - a^2$ is

(a) ab (b) 2ab (c) – ab (d) none

SOLUTION. $\frac{c}{\sin 110^o} = \frac{b}{\sin 15^o} = \frac{a}{\sin 55^o} = k$



Now, $c^2 - a^2 = k^2 \sin^2 110^o - k^2 \sin^2 55^o$

$= k^2 (\sin 110^o + \sin 55^o)(\sin 110^o - \sin 55^o)$

$= k^2 \left(2\sin\frac{165^o}{2}\cos\frac{55^o}{2}\right)\left(2\cos\frac{165^o}{2}\sin\frac{55^o}{2}\right)$

$= k^2 \sin 165^o \sin 55^o$

$= (k\sin 15^o)(k \sin 55^o) = ab.$

Hence (c) is correct answer.

TRIGONOMETRY

1. The equation $\sin^4x - 2\cos^2x + a^2 = 0$ is solvable for x∈R
   (a) – √2 ≤ a ≤ √2 (b) – √3 ≤ a ≤ √3
   (c) –1 ≤ a ≤ 1 (d) none
2. If $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$, then x =
   (a) 2 (b) 1 (c) – 2 (d) –1
3. If tan(θ + α) tan(θ – α) = 1 for all α∈ R, then θ is equal to (n ∈ I)
   (a) 2nπ ± π/4 (b) nπ ± π/4
   (c) 2nπ + π/2 (d) 2nπ – π/4
4. If y = $\frac{2\sin\alpha}{1+\cos\alpha+\sin\alpha}$, then $\frac{1-\cos\alpha+\sin\alpha}{1+\sin\alpha}$ is equal to
   (a) 1 + y (b) 1 – y (c) 1/y (d) y
5. $\sin^{-1} x + \sin^{-1} y + \sin^{-1} z = \pi/2$ then $x^2 + y^2 + z^2 + 2xyz$ is equal to
   (a) – 1 (b) 0 (c) 1 (d) none
6. If in a ΔABC, cosA + 2cosB + cosC = 2, then a, b, c are in
   (a) H.P (b) G.P (c) A.P (d) none
7. The equation $\sin^{-1} x - \cos^{-1}x = \cos^{-1}(\sqrt{3}/2)$ has
   (a) A unique solution (b) No solution
   (c) Infinitely many solution (d) none
8. $\frac{x}{\cos\theta} = \frac{y}{\cos\left(\theta+\frac{2\pi}{3}\right)} = \frac{z}{\cos\left(\theta-\frac{\pi}{3}\right)}$ then x + y + z =
   (a) – 1 (b) 1 (c) 0 (d) none
9. $\sin\{\sin^{-1}(1/5) + \cos^{-1}x\} = 1$ then x is equal to
   (a) 1/5 (b) 1 (c) 0 (d) 4/5
10. If a = 2, b = 3, c = 5 in a ΔABC, then C =
    (a) π/6 (b) π/3 (c) π/2 (d) none
11. The number of solutions of the equation |cotx | = cotx + 1/sinx, 0 ≤ x ≤ 2 π is
    (a) 1 (b) 2 (c) 3 (d) none
12. If $\sin^2\theta + 3\cos\theta - 2 = 0$ then $\cos^3\theta + \sec^3\theta$ is equation to
    (a) 18 (b) 9 (c) 4 (d) ¼
13. If $\tan^2\theta = 2\tan^2\phi + 1$, then $\cos2\theta + \sin^2\phi$ is equal to
    (a) 0 (b) $2\sin^2\phi$ (c) – cos2ϕ (d) none
14. The maximum value of sinx + cosx is
    (a) 1 (b) 2 (c) √2 (d) 1/√2
15. If 3 sinα = ½ sin(α + 2β), then $\frac{\tan(\alpha+\beta)}{\tan\beta}$ =
    (a) 5/3 (b) 1/3 (c) 3 (d) 3/5
16. If 3sin2θ = 2sin3θ, 0 < θ < π, then the value of sinθ =
    (a) 0 (b) √15/4 (c) –1/4 (d) none
17. $\sin\{\sin^{-1}(\sqrt{3}/5) + \cos^{-1}(\sqrt{3}/5)\}$ is equal to
    (a) π/2 (b) 1 (c) 0 (d) none
18. The value of the expression sinθ + cosθ lies between–2 and 2 both inclusive0 an √2 both inclusive– √2 and √2 both inclusive0 and 2 both inclusive.
19. The value of the expression 3cosθ + 4sinθ lies between
    (a) [– 7, 7] (b) [– 25, 25] (c) [– 1, 1] (d) [– 5, 5]
20. The minimum value of $\sin^6\theta + \cos^6\theta$ is
    (a) 0 (b) 1 (c) ¼ (d) none
21. If $x = \sin^2\theta \cos\theta$ and $y = \cos^2\theta \sin\theta$, then
    (a) $(x^2y)^{2/3}$ $(xy^2)^{2/3} = 1$
    (b) $(x^2/y)^{2/3} + (y^2/x)^{2/3} = 1$
    (c) $x^2 + y^2 = x^2y^2$
    (d) None
22. If x = secθ – tanθ and y = cosecθ + cotθ, then xy + 1 is equal to
    (a) x + y (b) x – y (c) 2x + y (d) y –x.
23. If secθ = x +1/4 x, x ∈ R, x ≠ 0, then the value of secθ + tanθ is
    (a) 2x (b) 1/2x (c) 2x or 1/2x (d) none
24. If tanθ = p/q, then the value of $\frac{p\sin\theta - q\cos\theta}{p\sin\theta + q\cos\theta}$ is
    (a) $\frac{p^2-q^2}{p^2+q^2}$ (b) $\frac{p^2+q^2}{p^2-q^2}$ (c) 0 (d) none
25. If x = a cosθ + b sinθ and y = a sinθ – b cosθ then $a^2 + b^2$ is equal to
    (a) $x^2 - y^2$ (b) $x^2 + y^2$ (c) $(x + y)^2$ (d) none
26. The minimum value of $\sec^2\theta + \cos^2\theta$ is
    (a) 1 (b) 0 (c) 2 (d) none
27. tanx is periodic with period
    (a) π/2 (b) π (c) 2π (d) 3π/2
28. If 0 < θ < π/4 then sec2θ – tan2θ is equal to
    (a) tan(π/4 + θ) (b) tan(θ – π/4)
    (c) tan(π/4 – θ) (d) none

29. The most general value of satisfying the equations $\sin\theta = \sin x$ and $\cos\theta = \cos x$ is
(a) $2n\pi + \alpha$ (b) $2n\pi - \alpha$ (c) $n\pi + \alpha$ (d) $n\pi - \alpha$
30. If $x/a \cos\theta + y/b \sin\theta + 1 = 0$ and $x/a \sin\theta - y/b \cos\theta - 1 = 0$ then$x^2/a^2 + y^2/b^2$ is equal to
(a) 2 (b) 0 (c) –2 (d) 1.
31. All solution of the equation $2\sin\theta + \tan\theta = 0$ are obtained by taking all $2n\theta\pi + 2\pi/3$ and $m\pi n\pi$ and $2m\pi \pm 2\pi/3 n\pi$ and $m\pi \pm \pi/3 n\pi$ and $2m\pi + \pi/3$
32. If $\cos(A - B) = 3/5$ and $\tan A \tan B = 2$ then
(a) $\cos A \cos B = 1/2$
(b) $\sin A \sin B = -2/5$
(c) $\cos(A + B) = 1/5$
(d) $\sin A \sin B = 4/5$
33. The general solution of$\sin^2\theta \sec\theta + \sqrt{3}\tan\theta = 0$ is
(a) $\theta = n\pi + (-1)^{n+1}\pi/3, \theta = n; n \in I$
(b) $\theta = n\pi, n \in I$
(c) $\theta = n\pi/2, n \in I$
(d) $\theta = n\pi + (-1)^{n+1} \pi/3, n \in I$
34. The equation $2\cos\phi - 3\sin\phi = k$ has real solution in ϕ when
(a) $k = -4$ (b) $|k| \le \sqrt{13}$ (c) $|k| < 5$ (d) $|k| = 5$.
35. The value of θ satisfying $\cos^2\theta + \sin\theta + 1 = 0$ and $0 \le \theta \le 2\pi$ lies in the interval
(a) $(-\pi/4, \pi/4)$ (b) $(5\pi/4, 7\pi/4)$
(c) $(\pi/4, \pi)$ (d) $(3\pi/4, \pi/4)$
36. If the equation $x - \cos\theta = \sin\theta$ has a real solution in θ. Than
(a) $x \le -\sqrt{2}$ (b) $x \ge \sqrt{2}$
(c) $|x| \ge \sqrt{2}$ (d) $x \le \sqrt{2}$
37. $\cos 2x + 2\cos x$ is
(a) $-3/2$ (b) $\ge -3/2$ (c) $\le 3/2$ (d) $< 3/2$
38. If $\cos A = 3/2$, then $32\sin(A/2)\sin(5A/2)$ is equal to
(a) 11 (b) – 11 (c) 8 (d) none
39. If α is a root of $25\cos^2\theta + 5\cos\theta - 12 = 0, \pi/2 < \alpha < \pi$, Then $\sin 2\alpha$ is equal to
(a) 24/25 (b) –24/25 (c) 25/24 (d) none
40. The maximum value of $12\sin\theta - 9\sin^2\theta$ is
(a) 3 (b) 4 (c) 5 (d) none
41. If $1 + \sin x + \sin^2 x + \ldots\ldots$to $\infty = 4 + 2\sqrt{3}, 0 < x < \pi$, then x is equal to
(a) $\pi/6$ (b) $\pi/4$ (c) $\pi/3$ or $2\pi/3$ (d) none
42. If $x = \sin^8\theta + \cos^{14}\theta$ then
(a) $x \ge 1$ (b) $0 \le x \le 1$ (c) $0 < x \le 1$ (d) none
43. If $\tan A = a/a+1$ and $\tan B = 1/2a + 1$then a value of A + B is
(a) 90º (b) 135º (c) 45º (d) none
44. If $\sin\theta = 2t/1 + t^2$ and t lies in the second quadrant, then $\cos\theta$ is equal to
(a) $\frac{1-t^2}{1+t^2}$ (b) $\frac{t^2-1}{1+t^2}$ (c) $\frac{-|1-t^2}{1+t^2}$ (d) none
45. The number of solution of the equation $\sin(\pi x/2\sqrt{3}) = x^2 - 2\sqrt{3}x + 4$ is
(a) 0 (b) 2 (c) 1 (d) none
46. The general solution of the equation $\tan 2x + \tan x + \tan x.\tan 2x = 1$is $(n \in I)$
(a) $n\pi + \pi/4$ (b) $n\pi/3 + \pi/12$
(c) $n\pi/3 \pm \pi/12$ (d) none
47. If $\tan \alpha = 1/7$ and $\tan \beta = 1/3$, than $\cos 2\alpha$ is equal to
(a) $\sin 2\beta$ (b) $\cos 2\beta$ (c) $\sin 4\beta$ (d) none
48. General solution of the equation $\tan\theta\tan 2\theta = 1$ is given by
(a) $(2n + 1) \pi/6, n \in I$
(b) $n\pi + \pi/6, n \in I$
(c) $n\pi - \pi/6, n \in I$
(d) $n\pi \pm \pi/6, n \in I$
49. If $f(x) = \sin^6 x + \cos^6 x$, then the period of the function f(x) is
(a) $\pi/3$ (b) $\pi/2$ (c) π (d) 2π
50. $8\sin(x/8).\cos(x/2).\cos(x/4).\cos(x/8)$ is equal to
(a) 8sinx (b) sinx (c) cosx (d) 8cosx
51. The minimum value of $\sec^2\theta + \csc^2\theta$ is
(a) 2 (b) 4 (c) 0 (d) 1
52. The number of points of intersection of $2y = 1$ and $y = \cos x, -\pi/2 \le \pi/2$ is
(a) 1 (b) 2 (c) 3 (d) 4
53. If $\cos\theta = \frac{\cos\alpha - \cos\beta}{1 - \cos\alpha \cos\beta}$ then $\tan\theta/2 =$
(a) $\pm \tan\alpha/2 \tan \beta/2$
(b) $\pm \tan \alpha/2 \cot \beta/2$
(c) $\pm \tan \beta/2 \cot \alpha/2$
(d) none
54. If $\tan\beta = \cos\theta \tan \alpha$ then $\tan^2(\theta/2) =$ (a) $\frac{\sin(\alpha+\beta)}{\sin(\alpha-\beta)}$
(b) $\frac{\sin(\alpha-\beta)}{\sin(\alpha+\beta)}$ (c) $\frac{\cos(\alpha-\beta)}{\cos(\alpha+\beta)}$ (d) $\frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)}$
55. If $\tan\theta = -1$ and $\cos\theta = -1/\sqrt{2}$, than θ is $(n \in 1)$ (a) $n\pi - \pi/4$ (b) $2n\pi \pm \pi/4$ (c) $2n\pi + 7\pi/4$ (d) none
56. If $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta\tan 4\theta\tan 7\theta$, then θ is $(n \in I)$
(a) $n\pi/4$ (b) $n\pi/7$ (c) $n\pi$ (d) none
57. If $\tan\theta + \tan 4\theta + \tan 7\theta = \tan\theta \tan 4\theta \tan 7\theta$, then θ is $(n \in I)$
(a) $n\pi/4$ (b) $n\pi/7$ (c) $n\pi$ (d) $n\pi/12$
58. If $\tan 3x = \cot x$ then x is $(n \in I)$
(a) $(2n + 1)\pi/8$ (b) $(2n + 1)\pi/6$
(c) $(2n + 1)\pi/4$ (d) none
59. If $\tan(\frac{\pi}{2}\sin\theta) = \cot(\frac{\pi}{2}\cot\theta)$, then $\sin\theta + \cos\theta =$
(a) 0 (b) 1 (c) – 1 (d) 1 or –1
60. If $\sin(\frac{\pi}{4}\cot\theta) = \cot\frac{\pi}{4}\tan\theta$ then θ is $(n \in I)$
(a) $n\pi + \pi/4$ (b) $2n\pi \pm \pi/4$
(c) $2n\pi - \pi/4$ (d) $n\pi - \pi/4$
61. An are of a circle of radius 77 cm subtends an angle of the arc is
(a) 121/9 cm (b) 770 cm (c) 121 cm (d) none
62. The value of $\tan 5A - \tan 3A - \tan 2A$ is equal to
(a) $\tan 5A \tan 3A \tan 2A$

(b) – tan5A tan3A tan2A
(c) tan2A tan3A – tan3A
(d) tan5A tan2Atan5A

63. The maximum value of $\sin(x + \frac{\pi}{6}) + \cos(x + \frac{\pi}{6})$ in the interval [0, π/2] is attained at x =
(a) π/6 (b) π/3 (c) π/12 (d) π/2

64. If $\frac{1}{2}(x + \frac{1}{x}) = \cos\theta$, then $\frac{1}{2}(x^3 + \frac{1}{x^3})$ is equal to
(a) $\cos^3\theta$ (b) $\cos3\theta$ (c) $\cos\theta - \cos^3\theta$ (d) none

65. If x ∈ (0, π/2)m then sin5x + sin3x + sinx = 0 is true for
(a) $x = \frac{\pi}{6}$ (b) $x = \frac{\pi}{12}$ (c) $x = \frac{\pi}{3}$ (d) $x = \frac{\pi}{9}$

66. If $\frac{\pi}{2} \le x \le \pi$ and $81^{\sin^2x} + 81^{\cos^2x} = 30$, then x is equal to
(a) π/3 or π/6 (b) 2π/3 or 5π/6
(c) π/6 or 5π/6 (d) none

67. The equation a sinx + cos2x = 2a –7 has a solution in x ∈ R if(a) a > 6 (b) a < 2(c) 2 ≤ a ≤ 6
(d) none

68. If secθ – tanθ = ½, then θ lies in
(a) the first quadrant
(b) the second quadrant
(c) the third quadrant
(d) the fourth quadrant

69. If A = 130º and x = sinA + cosA then
(a) x > 0 (b) x < 0 (c) x = 0 (d) x ≤ 0

70. The general solution of the equation
sinx – 3sin2x+ sin3x = cosx – 3cos2x + cos3x is
(a) $n\pi + \frac{\pi}{8}$, n ∈ I
(b) $\frac{n\pi}{2} + \frac{\pi}{8}$, n ∈ I
(c) $(-)^n (\frac{n\pi}{2}) + \frac{\pi}{8}$, n ∈ I
(d) $2n\pi + \cos^{-}(\frac{3}{2})$, n ∈ I

71. $1 - \tan\frac{A}{2}\tan\frac{B}{2} =$
(a) $\frac{2a}{a+c+c}$ (b) $\frac{2c}{a+b+c}$ (c) $\frac{2b}{a+b+c}$ (d) none

72. The equation $\sin^6x + \cos^6x = a$ has a real solution in x if
(a) $\frac{1}{2} \le a \le 1$ (b) $\frac{1}{4} \le a \le 1$
(c) $-1 \le a \le 1$ (d) $0 \le a \le \frac{1}{2}$

73. If $\sin x + \sin^2x = 1$, then $\cos^2x + \cos^4x$ is equal to
(a) 1 (b) –1 (c) 2 (d) 0

74. If sinx + cosx = a, then | sinx – cosx | equals
(a) $\sqrt{2a^2}$ (b) $\sqrt{2 + a^2}$ (c) $\sqrt{a^2 - 2}$ (d) none

75. If 3sinθ + 4 cosθ = 5, then the value of 3cosθ – sinθ is equal to
(a) 0 (b) – 5 (c) 5 (d) none

76. The value of $\sum_{x=1}^{360} \sin(x^0)$ is equal to
(a) 1 (b) –1 (c) 0 (d) none

77. 1 + cos2x + cos4x + cos6x =
(a) 2 cos x cos 2x cos 3x
(b) 4 sin x cos 2x cos 3x
(c) 4 cos x cos 2x cos 3x
(d) None of these

78. The greatest angle of a cyclic quadrilateral is 3 time the last. The circular measure of the leas angle is
(a) 45º (b) π/4 (c) π/3 (d) none

79. If sinθ = sinα, then the value of sinθ/3 is
(a) sin(α/3) (b) $\sin(\frac{2\pi - \alpha}{3})$
(c) $\sin(\frac{\pi + \alpha}{3})$ (d) sin (–α/3)

80. Circular measure of an angle of 1 radian is.
(a) 90 (b) π/2 (c) 1 (d) none

81. If sinθ = –1/2 and cosθ = √3/2, then the quadrant in which θ lies is
(a) I (b) II (c) III (d) IV

82. If 8 θ = π, then cos7θ + cosθ is equal to
(a) 1 (b) 0 (c) –1 (d) none

83. If sinα + sinβ + sinγ = 3, then he value of cosα + cosβ + cosγ =
(a) 0 (b) 1 (c) 2 (d) 3

84. The minimum value of $9\tan^2\theta + \cot^2\theta$ is equal to
(a) 13 (b) 9 (c) 6 (d) 12

85. If ABC is a right triangle with right angle at C, then tanA + tanB is equal to
(a) c^2/ab (b) a + b (c) a^2/bc (d) none

86. In a ΔABC, a = 13 cm, b = 12 cm and c = 5 cm. The distance of A from BC is
(a) $\frac{25}{13}$ cm (b) $\frac{60}{13}$ cm (c) $\frac{65}{12}$ cm (d) $\frac{144}{13}$ cm

87. If in a triangle ABC sin2A + sin2B = sin2C then the triangle is
(a) isosceles (b) right angled
(c) equilateral (d) none

88. In a ΔABC , a : b : c :: 4 : 5 : 6, then 3A + B is equal to
(a) 4C (b) 2π (c) π (d) π – C

89. Two angles of triangle are $\cot^{-1}2$ and $\tan^{-1} 1/3$. The third angle of the triangle is
(a) 2π/3 (b) π/2 (c) 5π/6 (d) 3π/4

90. If H is the orthocenter of a triangle is ABC, then AH equals
(a) acotB (b) acotA (c) bcotA (d) ccotA

91. In a triangle ABC, ∠ A = 45º, then the value of (1 + cotB)(1 + cotC) is equal to
(a) 0 (b) 1 (c) –1 (d) 2

92. In any triangle ABC, ∑ a sin(b – C) is equal to
(a) 0 (b) 3abc (c) ab + bc + ca (d) 3(a + b + c)

93. In a ΔABC, a = 2, b = 3 and sinA = 2/3, then B is equal to
(a) 30º (b) 60º (c) 90º (d) 120º

94. The angle of a triangle are as 1 : 2 :7, the ratio of greatest side to least side is
(a) (√2+ 1) : (√2 –1)
(b) (√3 + 1) : (√3 – 1)
(c) (√5 + 1) : (√5 – 1)
(d) (2 + √3) : (2 – √3)

95. In a ΔABC,
(b + c)cosA + (c + a)cosB+ (a + b)cosC is equal to

(a) 0 (b) a + b + c (c) Rr (d) none

96. In a triangle ABC $\tan\frac{A+B}{2}\cot\frac{A-B}{2}$ is equal to
(a) $\frac{a-b}{a+b}$ (b) $\frac{a+b}{c}$ (c) $\frac{a+b}{a-b}$ (d) none

97. If a = 4, b = 3 A = 60º, then c is a root
(a) $x^2 - 3x - 7 = 0$
(b) $x^2 - 3x + 7 = 0$
(c) $x^2 - 3x + 7 = 0$
(d) $x^2 - 3x - 7 = 0$

98. If the sides of a triangle be $2x + 1$, $x^2 - 1$ and $x^2 + x + 1$, then the greatest tangle is (a) 90º (b) 150º (c) 135º (d) 120º

99. If in a triangle ABC, $a\cos^2(C/2) + \cos^2(A/2) = 3b/2$ Then its sides a, b, c are in (a) AP (b) GP (c) HP (d) none

100. If in a ΔABC, acosA = bcosB, then the triangle is
(a) right angled (b) equilateral
(c) isosceles (d) either isosceles or right angled

101. If $\cos A + \cos B = 4\sin^2 C/2$, then the sides of the triangle ABC are in
(a) AP (b) GP (c) HP (d) none

102. In any ΔABC, if C = 90º, then tanB/2 is equal to
(a) $\sqrt{\frac{c-a}{c+a}}$ (b) $\sqrt{\frac{a-c}{a+c}}$ (c) $\sqrt{\frac{c+a}{c-a}}$ (d) none

103. $\cot^{-1} 21 + \cot^{-1} 13 + \cot^{-1}(-8)$ is equal to
(a) 0 (b) $\cot^{-1}26$ (c) π (d) none

104. $\cos^{-1}(\cos(-\pi/3))$ is equal to
(a) –π/3 (b) π/3 (c) 2π/3 (d) none

105. The value if $\sin(2\cos^{-1}(-3/5))$ is
(a) 24/25 (b) –24/25 (c) 7/25 (d) none

106. $\tan(\sin^{-1}x)$ is equal to
(a) $x/\sqrt{(1-x^2)}$ (b) $-x/\sqrt{(1-x^2)}$
(c) $|x|/\sqrt{(1-x^2)}$ (d) none

107. $\tan\{1/2\sin^{-1}(2x/1 + x^2) + 1/2\cos^{-1}(\frac{1-y^2}{1+y^2})\}$ is equal to
(a) $\frac{x-y}{1+xy}$ (b) $\frac{x+y}{1-xy}$ (c) $\frac{1}{2}(\frac{x+y}{1-xy})$ (d) $\frac{1}{2}(\frac{x-y}{1+xy})$

108. If $\sin^{-1} x = \theta + \beta$ $\sin^{-1}y = \theta - \beta$, then $1 + xy =$
(a) $\sin^2\theta + \sin^2\beta$ (b) $\sin^2\theta + \cos^2\beta$
(c) $\cos^2\theta + \cos^2\beta$ (d) none

109. The value of $\cos\{\sin^{-1}(\frac{1-\sqrt{x}}{1+\sqrt{x}}) + \sec^{-1}(\frac{1+\sqrt{x}}{1-\sqrt{x}})\}$ x > 0, x ≠ 1 is
(a) 1 (b) 0 (c) –1 (d) none

110. If $\cos(2\sin^{-1}x)$ 1/9 then x =
(a) 2/3 (b) –2/3 (c) ±2/3 (d) none

111. If $\tan^{-1}2x + \tan^{-1}3x = n\pi + 3\pi/4$, n ∈ I, then x =
(a) 1, –1/6 (b) ½ , 1/3 (c) 4, 5 (d) none

112. If | a | < 1, | b | < 1 and $\sin^{-1}(\frac{2a}{1+a^2}) + \sin^{-1}(\frac{2b}{1+b^2}) = 2\tan^{-1}x$, then x =
(a) 2a + 2b (b) a + b (c) $\frac{a+b}{a-b}$ (d) $\frac{a-b}{1+ab}$

113. If $\tan^{-1}\frac{x-1}{x+1} + \tan^{-1}\frac{2x-1}{2x+1} = \tan^{-1}(23/36)$ then x =
(a) 4/3 (b) –3/8 (c) 4/3 or –3/8 (d) none

114. The value of $\tan^{-1}(a/b) - \tan^{-1}(\frac{a-b}{a+b})$ is (a, b > 0)
(a) –π/4 (b) π/2 (c) –π/2 (d) π/4

115. If x, y, z ∈ R and x + y + z = xyz, then a value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is
(a) π (b) π/2 (c) 3π/2 (d) none

116. If x , y, z ∈ R and xy + yz + zx =1, then a value of $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z$ is
(a) π/2 (b) 0 (c) π (d) none

117. $2\cos^{-1}x = \cos^{-1}(2x^2 - 1)$ holds true for all
(a) |x| ≤ 1 (b) 0 ≤ x ≤ 1 (c) |x| < ½ (d) none

118. If $\sin^{-1}(5/x) + \sin^{-1}(12/x) = \pi/2$, then the value of x is equal to
(a) – 13 (b) ± 13 (c) 13 (d) none

119. The relation $\text{cosec}^{-1}(\frac{x^2+1}{2x}) = 2\cot^{-1}x$ is valid for
(a) x ≥ 1 (b) x ≥ 0 (c) | x | ≥ 1 (d) none

120. The number of solution of the equation $\sin^{-1}x - \cos^{-1}x = \sin^{-1}(1/2)$ is
(a) 1 (b) 2 (c) 3 (d) infinite

121. If solution of the equation $\cos^{-1}(\sqrt{3}x) + \cos^{-1}x = \pi/2$ is given by
(a) ± ½ (b) –1/2 (c) ½ (d) none

122. If $\sin^{-1}(1 - x) - 2\sin^{-1}x = \pi/2$, then x is equal to
(a) 0 (b) 1 (c) 0 or 1 (d) none

123. The number of solution of the equation $\cos^{-1}(1 - x) - 2\cos^{-1}x = \pi/2$ is
(a) two (b) one (c) more then one (d) none

124. For x ≥ 1, the value of $2\tan^{-1}x + \sin^{-1}(\frac{2x}{1+x^2})$ is
(a) $4\tan^{-1}x$ (b) 0 (c) π (d) none

125. The value of $\tan(\cos^{-1}(4/5) + \tan^{-1}(2/3)$ is equal to
(a) 6/17 (b) 16/7 (c) 7/16 (d) none

126. If tanθ + secθ = √3, 0 < π, then θ is equal to
(a) π/2 (b) π/6 (c) 2π/3 (d) 5π/6

127. The equation $(\cos p - 1)x^2 + (\cos p)x + \sin p = 0$, where x is a variable, had real roots. Then the interval of p may be any one of the following :
(a) (0, 2π) (b) (–π, 0) (c) (–π/2, π/2) (d) (0, π)

128. In a triangle ABC, a = 13, b = 14, c = 15, r =
(a) 4 (b) 8 (c) 2 (d) 6

129. If sinθ + cosecθ = 2, then $\sin^2\theta + \text{cosec}^2\theta =$
(a) 1 (b) 4 (c) 2 (d) none

130. If $A = \sin^2\theta + \cos^4\theta$ then for all real values of θ
(a) 1 ≤ A ≤ 2 (b) ¾ ≤ A ≤ 1
(c) 13/16 ≤ A ≤ 1 (d) ¾ ≤ A ≤ 13/16.

131. The general solution of tan3x = 1 is (n ∈ I)
(a) nπ + π/4 (b) $\frac{n\pi}{3} + \frac{\pi}{12}$
(c) nπ (d) nπ ± π/4

132. The value if $\sin\frac{\pi}{14}\sin\frac{3\pi}{14}\sin\frac{5\pi}{14}\sin\frac{9x}{14}\sin\frac{11\pi}{14}\sin\frac{13\pi}{14}$ is
(a) 1/16 (b) 1/64 (c) 1/128 (d) none

133. The maximum value of $\sin(x + \frac{\pi}{6}) + \cos(x + \frac{\pi}{6})$ in the interval (0,π/2) is attained at
(a) π/12 (b) π/6 (c) π/3 (d) π/2

134. The solution of tan2θ tanθ = 1 is

(a) $\pi/3$ (b) $(6n \pm 1)\pi/6$
(c) $(4n \pm 1)\pi/6$ (d) $(2n + 1)\pi/6$

135. $\sin(180 + \phi) \sin(180 - \phi) \operatorname{cosec}^2\phi$ is equal to
(a) 1 (b) –1 (c) 0 (d) none

136. The general solution of the equation $\sin x + \cos x = 2$ is
(a) $n\pi$, $n \in 1$ (b) $(2n + 1)\pi/2$, $n \in 1$
(c) $2n\pi + \pi/2$ or $2n\pi$, $n \in I$ (d) none

137. A solution of the equation $\tan^{-1}(1 + x) + \tan^{-1}(1 - x) = \pi/2$ is
(a) $x = 1$ (b) $x = -1$ (c) $x = 0$ (d) $x = \pi$

138. $\tan(\cos^{-1}x)$ is equal to
(a) $\sqrt{(1 + x^2)}/x$ (b) $x/1 + x^2$
(c) $\sqrt{(1 + x^2)}$ (d) $\sqrt{1 - x^2}$

139. The general value of θ satisfying the equation $2\sin^2\theta - 3\sin\theta - 2 = 0$ is
(a) $n\pi + (-1)^n\pi/2$ (b) $n\pi + (-1)^n\pi/2$
(c) $n\pi + (-1)^n 5\pi/6$ (d) $n\pi + (-1)^{n+1}\pi/6$

140. In a triangle ABC, if a, b, c are in A.P. then, $\tan\frac{A}{2}$, $\tan\frac{B}{2}$, $\tan\frac{C}{2}$ are in
(a) AP (b) GP (c) HP (d) none

141. In a ΔABC if $\frac{\cos A}{a} = \frac{\cos B}{b} = \frac{\cos C}{c}$ and the side a = 2, then area of the triangle is
(a) 1 (b) 2 (c) $\sqrt{3}/2$ (d) $\sqrt{3}$

142. In a ΔABC, a = 4, b = 3 and A = 60º, then c is a root of the equation.
(a) $c^2 - 3c - 7 = 0$ (b) $c^2 + 3c + 7 = 0$
(c) $c^2 - 3c + 7 = 0$ (d) $c^2 + 3c - 7 = 0$

143. If $x = y \cos\frac{2\pi}{3} = z \cos\frac{4\pi}{3}$, then $xy + yz + zx =$
(a) –1 (b) 0 (c) 1 (d) 2

144. If $\sin\theta_2 + \sin\theta_2 + \sin\theta_3 = 3$ then $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 =$
(a) 3 (b) 2 (c) 1 (d) 0

145. If $\sin\alpha = \sin\beta$ and $\cos\alpha = \cos\beta$, then
(a) $\sin\frac{\alpha + \beta}{2} = 0$ (b) $\cos\frac{\alpha - \beta}{2} = 0$
(c) $\sin\frac{\alpha - \beta}{2} = 0$ (d) $\cos\frac{\alpha - \beta}{2} = 0$.

146. If $\sin\alpha + \sin\beta = 0 = \cos\alpha + \cos\beta$, then $\cos2\alpha + \cos2\beta =$
(a) $-2\sin(\alpha + \beta)$ (b) $-2\cos(\alpha + \beta)$
(c) $2\sin(\alpha + \beta)$ (d) $2\cos(\alpha + \beta)$

147. If $\sin^{-1}x + \sin^{-1}y = 2\pi/3$, then $\cos^{-1}x + \cos^{-1}y =$
(a) $2\pi/3$ (b) $\pi/3$ (c) $\pi/6$ (d) π

148. The value if the expression tan1º tan2º tan3º……….tan89º is equal to
(a) 0 (b) not defined (c) 1 (d) ∞

149. The principal value of $\sin^{-1}(\sin\frac{2\pi}{3})$ is
(a) $4\pi/3$ (b) $2\pi/3$ (c) $\pi/3$ (d) $-2\pi/3$

150. $4\tan^{-1}(1/5) - \tan^{-1}(1/239) =$
(a) π (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/4$

151. If $\sin x - \cos x = \sqrt{2}$, then x = (n is any integer)(a) $2n\pi$ (b) $n\pi$(c) $(2n + 1)\pi$
(d) $2n\pi + 3\pi/4$

152. If $\frac{\cos A}{3} = \frac{\cos B}{4} = \frac{1}{5}$, $\frac{\pi}{2} < A < 0$, $-\frac{\pi}{2} < 0$, then value of $2\sin A + 4\sin B =$
(a) 4 (b) 2 (c) – 4 (d) 0

153. The general value of θ satisfying $\sin\theta = -1/2$ and $\tan\theta = 1/\sqrt{3}$ is
(a) $n\pi + \frac{\pi}{6}$, $n \in I$ (b) $n\pi + (-1)^n \frac{7\pi}{6}$, $n \in I$
(c) $2n\pi + \frac{7\pi}{6}$, $n \in I$ (d) $2n\pi + \frac{11\pi}{6}$, $n \in I$

154. The equation $\sin x - \frac{\pi}{2} + 1 = 0$ has no root in the interval
(a) $(0, \pi/2)$ (b) $(\pi/2, \pi)$ (c) $(\pi/2, 3\pi/2)$ (d) none

155. The number of values of x satisfying the condition $\sin x + \sin 5x = \sin 3x$ in the interval $[0, \pi]$ is
(a) 6 (b) 2 (c) 10 (d) 0

156. If $\cot^{-1}(\sqrt{\cos\alpha}) + \tan^{-1}(\sqrt{\cos\alpha}) = \mu$ then $\sin\mu$ is equal to
(a) $\tan^2\alpha$ (b) $\tan2\alpha$ (c) 1 (d) $\cot^2(\alpha/2)$

157. The greatest value of $f(x) = 2\sin x + \sin 2x$ on $[0, 3\pi/2]$ is given by
(a) 9/2 (b) 5/2 (c) $3\sqrt{3}/2$ (d) 3/2

158. If the area of ΔABC is $a^2 - (b - c)^2$, then tanA =
(a) –8/15 (b) 15/16 (c) 8/15 (d) 15/8

159. If $\cos^{-1}x + \sin^{-1}(x/2) = \pi/6$, then x =
(a) 0 (b) $1/\sqrt{2}$ (c) 1 (d) $\pm\sqrt{3}$

160. Solution of $\tan\theta + \tan4\theta + \tan7\theta = \tan\theta \tan4\theta \tan7\theta$ is given by $(n \in I)$
(a) $n\pi$ (b) $(2n+1)\pi/2$
(c) $n\pi + (-1)^n\pi/12$ (d) $n\pi/12$

161. If $\tan^{-1}x + \tan^{-1} 1/7 = \pi/4$, then x =
(a) ¾ (b) 6/7 (c) 4/3 (d) 7/6

162. $\tan^{-1}3 - \tan^{-1}2 =$
(a) $\tan^{-1}(3/2)$ (b) $\tan^{-1}(2/3)$
(c) $\tan^{-1}(1/7)$ (d) $\tan^{-1}(1/5)$

163. The general value of θ satisfying $\sin^2\theta + \sin\theta = 2$ is
(a) $n\pi + (-1)^n \pi/6$ (b) $n\pi + (-1)^n\pi/4$
(c) $n\pi + (-1)^n \pi/2$ (d) $n\pi + (-1)^n \pi$

164. The minimum value of $\sin\theta + \cos\theta$ is
(a) 0 (b) 1(c) –1 (d) $-\sqrt{2}$

165. The value of cos52º + cos68º + cos172º is equal to
(a) 0 (b) 2 (c) ½ (d) none

166. In a ΔABC if b + c = 3a, then $\cot\frac{B}{2} \cot\frac{C}{2}$ is equal to
(a) 1 (b) 2 (c) 3 (d) none

167. If $3\sin^{-1}(\frac{2x}{1+x^3}) - 4\cos^{-1}(\frac{1-x^2}{1+x^2}) + 2\tan^{-1}(\frac{2x}{1-x^2}) = \frac{\pi}{3}$, then x = (a) 1 (b) 2 (c) $1/\sqrt{3}$ (d) $1/\sqrt{2}$

168. tan70º – tan20º – 2tan 40º =
(a) 2tan20º (b) tan40º (c) 4tan10º (d) tan10º

169. If $\sin(\pi\cos\theta) = \cos(\pi\sin\theta)$, then $\sin2\theta =$
(a) $\pm 3/4$ (b) $\pm\sqrt{2}$ (c) $\pm 1/\sqrt{3}$ (d) ± ½

170. The value of x which satisfy the trigonometric equation $\tan^{-1}\left(\frac{x-1}{x-2}\right) + \tan^{-1}\left(\frac{x+1}{x+2}\right) = \frac{\pi}{4}$ are
(a) $1/\sqrt{2}$ (b) $\pm\sqrt{2}$ (c) $\pm ½$ (d) ± 2

171. If A + B = 45°, then (cotA –1) (cotB – 1) =
(a) 0 (b) 2 (c) 1 (d) 4

172. $\tan\theta + 2\tan 2\theta$ =
(a) $\text{cosec}\theta - 4\text{cosec}4\theta$ (b) $\cot\theta - 4\cot 4\theta$
(c) $\cos\theta - 4\cos 4\theta$ (d) $\sin\theta - 4\sin 4\theta$

173. The value of $\cos\frac{2\pi}{7} + \cos\frac{4\pi}{7} + \cos\frac{6\pi}{7} + \cos\frac{7\pi}{7}$ =
(a) 1 (b) – 1 (c) ½ (d) –3/2

174. If $\tan\theta\tan(\frac{\pi}{3} + \theta)\tan(\theta - \frac{\pi}{3}) = k\tan 3\theta$, then k is equal to
(a) 1 (b) 1/3 (c) 3 (d) none3

175. If the angles of triangle are 30° and 45° the included side is $(\sqrt{3} + 1)$ cm, then the area of the triangle is in cm²)
(a) $1/\sqrt{3} - 1$ (b) $\sqrt{3} + 1$
(c) $1/\sqrt{3} + 1$ (d) none

176. If $f(x) = \frac{1-x}{1+x}$, then f(x) =
(a) cosx (b) $\tan^2(x/2)$ (c) $\frac{1-\cos x}{1+\cos x}$ (d) x

177. Value of sin 47° + sin 61° – sin 11° – sin 25° is
(a) cos 7° (b) sin 7° (c) sin 61° (d) –sin 25°

178. If in a ΔABC, angle A, B, C are in AP, side a, b, c are in GP, then a^2, b^2, c^2 are in
(a) AP (b) HP (c) AP or HP (d) none

179. IF $\sin^{-1}\frac{3}{5} + \cos^{-1}\left(\frac{12}{13}\right) = \sin^{-1} C$, then C =
(a) 65/56 (b) 24/65 (c) 16/65 (d) 56/65

180. If $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$, then θ is equal to (n ≥ I)
(a) $2n\pi - \pi/6$ (b) $2n\pi - \pi/3$
(c) $2n\pi - \pi/12$, $2n\pi + 5\pi/12$ (d) $2n\pi \pm \pi/12$

181. If $\tan(\pi\cos\theta) = \cot(\pi\sin\theta)$, then the value of $\cos(\theta - \pi/4)$ =
(a) $1/2\sqrt{2}$ (b) $1/\sqrt{2}$ (c) $1/3\sqrt{2}$ (d) $1/4\sqrt{2}$

182. If $\cos^{-1}(x/a) + \cos^{-1}(y/b) = \alpha$, then $\frac{x^2}{a^2} - \frac{2xy}{ab}\cos\alpha + \frac{y^2}{b^2}$ =
(a) $\cos^2\alpha$ (b) $\tan^2\alpha$ (c) $\sin^2\alpha$ (d) $\cot^2\alpha$

183. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = \pi$, then $x\sqrt{(1-x^2)} + y\sqrt{(1-y^2)} + z\sqrt{(1-z^2)}$ =
(a) 2xyz (b) xyz (c) 4xyz (d)1/2 xyz

184. If A lies in the second quadrant and3tanA + 4 = 0, then the value of 2cotA – 5cosA + sinA is equal to
(a) 37/10 (b) 23/10 (c) –53/10 (d) none

185. If the period of the function f(x) = sin(x/n) is 4π, then n is equal to then x is equal to
(a) 1 (b) 4 (c) 8 (d) 2

186. If $A = \begin{bmatrix} 1 & a & a^2 \\ \cos(n-1)x & \cos nx & \cos(n+1)x \\ \sin(n-1)x & \sin nx & \sin(n+1)x \end{bmatrix}$ then det A is independent of
(a) a (b) x (c) n (d) none

187. If the sides of a triangle are $x^2 + x + 1$, $x^2 - 1$ and $2x + 1$, then the greatest angel is
(a) 90° (b) 135° (c) 115° (d) 105° (e) 120°

188. If $\cot(\alpha + \beta) = 0$, then $\sin(\alpha + 2\beta)$ =
(a) sinα (b) cosα (c) sinβ
(d) cos2β (e) sin2α

189. The value of $4\sin A\cos^3 A - 4\cos A\sin^3 A$ =
(a) cos2²A (b) sin3A (c) sin2A
(d) cos4A (e) sin4A

190. 190. If the solution for θ of cospθ + cosqθ = 0, p > 0 are in AP , then the numerically smallest common difference of AP is
(a) $\frac{\pi}{p+q}$ (b) $\frac{2\pi}{p+q}$ (c) $\frac{\pi}{2(p+q)}$
(d) $\frac{1}{p+q}$ (e) $\frac{1}{2(p+q)}$

191. If $\text{cosec}\theta = \frac{p+q}{p-q}$, then $\cot(\frac{\pi}{4} + \frac{\theta}{2})$ =
(a) $\sqrt{p/q}$ (b) $\sqrt{q/p}$ (c) $\sqrt{pq}$ (d)pq

192. $1 + \sin x + \sin^2 x + \ldots\ldots$to $\infty = 4 + 2\sqrt{3}$, if
(a) x = 2π/3 or π/3 (b) x = 7π/6
(c) x = π/6 (d) x = π/4

193. In a triangle, the lengths of two larger sides are 10 and 9 respectively. If the angles are in AP, then the third side can be
(a) $5 \pm \sqrt{6}$ (b) $5 - \sqrt{6}$ (c) $3\sqrt{3}$ (d) 5

194. The solution of the equation $\cos^2 x - 2\cos x = 4\sin x - \sin 2x$ $(0 \le x \le \pi)$ is
(a) $\pi - \cot^{-1}(1/2)$ (b) $\pi - \tan^{-1}2$
(c) $\pi + \tan^{-1}(-1/2)$ (d) none

195. $\sin\{\tan^{-1}\left(\frac{1-x^2}{2x}\right) + \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\}$ is equal to
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $1/\sqrt{2}$

196. If 0 < x < π/2, then the largest angle of a triangle whose sides are 1, sinx, cosx is
(a) π/2 (b) π/3 (c) π/2 – x (d) x

197. If $\cos^{-1}\sqrt{p} + \cos^{-1}\sqrt{1-p}$ then the value of q is
(a) 1 (b) $1/\sqrt{2}$ (c) 1/3 (d) ½

198. In a ΔABC, if a – 2x, b = 2y and ∠C = 120°, then the area of the triangle is
(a)xy (b) $xy\sqrt{3}$ (c) 3xy (d) 2xy

199. $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta$ is equal to
(a) 0 (b) 1 (c) –1 (d) none

200. If $\tan\frac{B-C}{2} = x\cot\frac{A}{2}$, then x =
(a) $\frac{c-a}{c+a}$ (b) $\frac{a-b}{a+b}$ (c) $\frac{b-c}{b+c}$ (d) none

201. If $\sqrt{3}\cos\theta + \sin\theta = \sqrt{2}$, then general value of θ is
(a) $n\pi + (-1)^n\pi/4$ (b) $(-1)^n\pi/4 - \pi/3$
(c) $n\pi + \pi/4 - \pi/3$ (d) $n\pi + (-1)^n\pi/4 - \pi/3$

202. The equation 4sinx + 3cosx = 6 has solution
(a) finite (b) infinite (c) one (d) none

203. The radius of the circle, whose are of length 1.5 cm makes an angle of ¾ radian at the centre is
(a) 10 cm (b) 20 cm (c) $11\frac{1}{4}$ cm (d) $22\frac{1}{2}$ cm

204. $\sec 50^\circ + \tan 50^\circ$ is equal to
(a) $\tan 20^\circ + \tan 50^\circ$ (b) $2\tan 20^\circ + \tan 50^\circ$
(c) $\tan 20^\circ + 2\tan 50^\circ$ (d) $2\tan 20^\circ + 2\tan 50^\circ$

205. The value of $\sin(\cot^{-1}x) =$
(a) $(1 + x^2)^{-1/2}$ (b) $(1 - x^2)^{1/2}$
(c) $(1 + x^2)^{1/2}$ (d) none

206. If $\cot(\cos^{-1}x) = \sec\{\tan^{-1}(\frac{a}{\sqrt{b^2-a^2}})\}$ then x equals
(a) $\frac{b}{\sqrt{2b^2-a^2}}$ (b) $\frac{a}{\sqrt{2b^2-a^2}}$
(c) $\frac{\sqrt{b^2-a^2}}{a}$ (d) $\frac{\sqrt{b^2-a^2}}{b}$

207. If $\tan\theta + \sec\theta = e^x$, then $\cos\theta$ equals
(a) $\frac{e^x+e^{-x}}{2}$ (b) $\frac{2}{e^x-e^{-x}}$
(c) $\frac{e^x-e^{-x}}{2}$ (d) $\frac{e^x-e^{-x}}{e^x+e^{-x}}$

208. If n = 1,2,3……, then
$\cos\alpha.\cos2\alpha.\cos2^2\alpha.\cos2^3\alpha \ldots.. \cos2^{n-1}\alpha$ is equal to
(a) $\frac{\sin 2n\alpha}{2n\sin\alpha}$ (b) $\frac{\sin 2^n\alpha}{2^n\sin 2^{n-1}\alpha}$
(c) $\frac{\sin 4^{n-1}\alpha}{4^{n-1}\sin\alpha}$ (d) $\frac{\sin 2^n\alpha}{2^n\sin\alpha}$

209. If $a = \pi/18$, then $\cos a + \cos 2a + \ldots.. + \cos 18a =$
(a) 0 (b) –1 (c) 1 (d) ±1

210. The number of value of θ in the interval $[-\pi, \pi]$ satisfying the equation $\cos\theta + \sin 2\theta = 0$ is
(a) 1 (b) 2 (c) 3 (d) 4

211. If $m\tan(\theta - 30^\circ) = n\tan(\theta + 120^\circ)$ then $\cos\theta$ equals
(a) $\frac{m+n}{m-n}$ (b) $\frac{m-n}{m+n}$ (c) $\frac{m-n}{2(m+n)}$ (d) $\frac{m+n}{2(m-n)}$

212. $\tan^{-1}(m/n) - \tan^{-1}(\frac{m-n}{m+n}) =$
(a) $\pi/4m$ (b) $\pi/2$ (c) $\pi/3$ (d) $\pi/8$

213. The solution set of $(5 + 4\cos\theta)(2\cos\theta + 1) = 0$ in the interval $[0, 2\pi]$ is
(a) $\{\pi/3, 2\pi/3\}$ (b) $\{\pi/3, \pi\}$
(c) $\{2\pi/3, 4\pi/3\}$ (d) $\{2\pi/3, 5\pi/3\}$

214. If $A + B + C = 270^\circ$, then
$\cos 2A + \cos 2B + \cos 2C + 4\sin A.\sin B.\sin C =$
(a) 0 (b) 1 (c) 2 (d) 3

215. If $\sin 6\theta = 32\cos^5\theta\sin\theta - 32\cos^3\theta\sin\theta + 3x$, then x
(a) $\cos\theta$ (b) $\cos 2\theta$ (c) $\sin\theta$ (d) $\sin 2\theta$

216. In a ΔABC, if $\begin{vmatrix} 1 & a & b \\ 1 & c & a \\ 1 & b & c \end{vmatrix} = 0$, then $\sin^2A + \sin^2B + \sin^2C =$
(a) 4/9 (b) 9/4 (c) $3\sqrt{3}$ (d) 1

217. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = 3\pi$, then $xy + yz + zx =$
(a) 1 (b) 0 (c) –3 (d) 3

218. If $\cos^{-1}(2x^2 - 1) + 2\cos^{-1}x = 360^\circ$, then the value of x must lie in the closed interval
(a) [0, 1] (b) [–1/2, –1/4]
(c) [–1, 0] (d) [–1, -1/2]

219. If in a triangle ABC $a\cos^2(C/2) + c\cos^2(A/2) = 3b/2$, then the sides a, b, and c
(a) satisfy a + b = c (b) are in AP
(c) are in GP (d) are in HP

220. In a triangle ABC, medians AB and BE are drawn. If AD = 4, ∠DAB = π/6 and ∠ABE = π/3, then the area of the triangle ABC is
(a) 64/3 (b) 8/3 (c) 32/3 (d) $32/3\sqrt{3}$

221. The sum of the radii of inscribed and circumscribed circles for an n sided regular polygon of side ‘a’ is
(a) $\frac{a}{4}\cot(\frac{\pi}{2\pi})$ (b) $a\cot(\frac{\pi}{n})$
(c) $\frac{a}{2}\cot(\frac{\pi}{2n})$ (d) $a\cot(\frac{\pi}{2n})$

222. The maximum value of $5\cos\theta + 3\cos(\theta + \pi/3) + 3$ is
(a) 5 (b) 10 (c) 11 (d) –1

223. Is $(\tan^{-1}x)^2 + (\cot^{-1}x)^2 = 5\pi^2/8$, then x =
(a) –1 (b) 1 (c) 0 (d) none

224. If $\tan\alpha = 1/5$, $\tan\beta = 1/239$, then the value of $\tan(4\alpha - \beta)$ is
(a) 0 (b) –1 (c) 1 (d) none

225. If a = 5, b = 4 and $\cos(\frac{A-B}{2}) = \frac{31}{32}$ then c =
(a) 1/6 (b) $\sqrt{41}$ (c) 6 (d) none

226. If $\sin^{-1}x + \sin^{-1}y + \sin^{-1}z = 3\pi/2$, then the value of $x^{100} + y^{100} + z^{100} - \frac{9}{x^{101}+y^{101}+z^{101}}$ is
(a) 0 (b) 1 (c) 2 (d) 3

227. If$|\cos\theta\{\sin\theta + \sqrt{\sin^2\theta + \sin^2\alpha}\}| \le k$ then the value of k is
(a) $\sqrt{1+\cos^2\alpha}$ (b) $\sqrt{1+\sin^2\alpha}$
(c) $\sqrt{2+\sin^2\alpha}$ (d) $\sqrt{2+\cos^2\alpha}$

228. If $e^{-\pi/2} < \theta < \pi/2$, then
(a) $\cos(\log\theta) > \log(\cos\theta)$
(b) $\cos(\log\theta) < \log(\cos\theta)$
(c) $\cos(\log\theta) = \log(\cos\theta)$
(d) $\cos(\log\theta) = 2/3\log(\cos\theta)$

229. If $3\tan^{-1}(\frac{1}{2+\sqrt{3}}) - \tan^{-1}1/2 = \tan^{-1}$ ½, then x =
(a) 1 (b) 3 (c) $\sqrt{3}$ (d) none

230. If $x = \sin(2\tan^{-1}2)$ and $y = \sin(1/2\tan^{-1} 4/3)$, then
(a) x < y (b) x > y
(c) $x + 1 = y^2$ (d) x > y and $y^2 = 1 - x$

ANSWERS

Trigonometry

1. a	45. a	89. d
2. a	46. d	90. b
3. b	47. c	91. d
4. d	48. d	92. a
5. c	49. b	93. c
6. c	50. b	94. c
7. a	51. b	95. b
8. c	52. b	96. c
9. a	53. b	97. a
10. d	54. b	98. d
11. a	55. c	99. a
12. a	56. d	100. d
13. a	57. d	101. a
14. c	58. a	102. a
15. c	59. d	103. c
16. b	60. a	104. b
17. b	61. a	105. b
18. c	62. a	106. a
19. d	63. c	107. b
20. c	64. b	108. b
21. b	65. c	109. b
22. d	66. b	110. c
23. c	67. c	111. a
24. a	68. a	112. c
25. b	69. a	113. a
26. c	70. b	114. d
27. b	71. b	115. a
28. c	72. b	116. a
29. d	73. a	117. b
30. a	74. a	118. c
31. b	75. a	119. a
32. a	76. c	120. a
33. b	77. c	121. c
34. b	78. b	122. a
35. b	79. a	123. b
36. d	80. c	124. c
37. b	81. d	125. d
38. a	82. b	126. b
39. b	83. a	127. d
40. b	84. d	128. a
41. c	85. a	129. c
42. c	86. b	130. b
43. c	87. b	131. b
44. c	88. c	132. b
133. a	165. a	197. d
134. b	166. b	198. b
135. b	167. c	199. b
136. d	168. c	200. c

137. c	169. a	201. d
138. a	170. a	202. d
139. d	171. b	203. b
140. c	172. b	204. c
141. d	173. d	205. a
142. a	174. d	206. a
143. b	175. a	207. b
144. d	176. a	208. d
145. c	177. a	209. b
146. b	178. a	210. d
147. b	179. d	211. d
148. c	180. c	212. a
149. c	181. a	213. c
150. d	182. c	214. b
151. d	183. a	215. d
152. c	184. b	216. b
153. c	185. d	217. d
154. d	186. c	218. c
155. a	187. e	219. b
156. c	188. a	220. d
157. c	189. b	221. c
158. c	190. d	222. b
159. c	191. d	223. a
160. d	192. a	224. c
161. a	193. a	225. c
162. c	194. c	226. a
163. c	195. b	227. b
164. d	196. a	228. a 229. b 230. a

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